

# An economic theory of academic competition : Dynamic incentives and endogenous accumulative advantages

Nicolas Carayol <sup>‡</sup>

BETA, CNRS, Université Louis Pasteur (Strasbourg 1)

and

ADIS, Université Paris Sud-Orsay (Paris 11)

This version : November 2005

(First version : March 2003)

**Abstract :** This paper provides a simple dynamic model of academic competition. Overlapping generations of researchers compete for first junior and next senior positions offered by universities which are also competing to hire the best scientists. Research positions have idiosyncratic associated productivities across universities and time because agents benefit from between-generations and within-universities positive externalities and from the accumulated reputation of the employing university. This feature is intended to capture the Matthew effect in science which designates the cumulative advantages affecting academic production. We derive equilibrium scientists' efforts, wages and long run endogenous accumulative advantage.

**Key words :** Matthew effect, accumulative advantages, biased sequential contests, academic competition.

**JEL classification :** C73, H00, J44, J41, J72.

<sup>‡</sup> Corresponding address. Nicolas Carayol, 61, avenue de la Forêt Noire, F-67085, Strasbourg Cedex.  
Tel : +33-390242069. Fax : +33-390242071. Email : <carayol@cournot.u-strasbg.fr>.

# 1 Introduction

The implicit and explicit rules of academic research stress a specific reward system in which *priority* is essential (Merton, 1957). The recognition of a scholar as the intellectual proprietor of the knowledge she produced increases her *credit* within the peer community. In turn scientific reputation translates into increased wages, more prestigious positions and other non monetary rewards (Dasgupta and David, 1994). The academic reward system appears to be fundamentally reputation-based which has two main implications on the provision of incentives across time (for a given scholar) and across scholars.

First reputation based incentives tend to distort the distribution of incentives during researchers' careers since the returns of research activity are usually delayed and spread over the remaining professional cycle. Scientists are thus essentially facing dynamic incentives (career concern). Because the expected returns of efforts are decreasing with the remaining activity period, the sharp decline of incentives in the late career is likely to overbalance the experience effect. Thus an inversed-U shape of scientific production distribution over the career-cycle is predicted. Several empirical studies using panel data of publication profiles have corroborated this statement (Weiss and Lillard, 1982 ; Levin and Stephan, 1992)<sup>1,2</sup>. The second consequence of such a reputation based reward system is that resources and means are not uniformly distributed across agents but tend to be concentrated in the hands of those who have more *credit*. Cumulative processes bias the academic competition providing some competitive advantage to the agents who have experienced the best early career accomplishments<sup>3</sup>. Analyses of the distribution of publication records among scholars<sup>4</sup> and time series analysis (Allison and Stewart, 1974) support the accumulative advantage hypothesis.

---

<sup>1</sup>Stephan and Levin (1997) show that the peak is often attained between 35 and 50 years old in the field of Biochemistry and Physiology. Diamond (1986) finds that the publication profiles of Berkeley University mathematicians decrease over the whole career. This specificity may be explained by the low experience effect in this discipline.

<sup>2</sup>Such results suggested improvements of human capital theory. Two technical solutions may be used. The first one consists in introducing human capital depreciation. Since it also generates a counter-factual decrease in wages, another solution has been suggested by Levin and Stephan (1991) who introduced a "puzzle solving" argument in academics objective function, namely the agents do not value only wages but also scientific production itself (or get other satisfactions from it but the wage).

<sup>3</sup>R. K. Merton (1968, 1988) gives the label of *Matthew effect* to the various accumulative advantages affecting the academic sphere. He refers to the quotation of the Gospel according to Saint Matthew : "for unto every one that hath shall be given, and he shall have abundance : but from him that hath not shall be taken away even that which he hath" (ancient English). The first evidencing of cumulative advantages in the academe is due to H. Zuckerman's Ph.D. thesis defended in 1965 (having R.K. Merton as a supervisor) and which was dedicated to studying Nobel Price laureates career paths. In the early emergence of this notion, both Merton and Zuckerman tend to limit the application of the notion to the symbolic mechanism according to which already reputed scholars gain more credit than less reputed ones from a co-authored paper or from a simultaneous discovery. The extension to various cumulative advantages comes later (e.g. in Merton, 1988).

<sup>4</sup>Such distribution is known to be highly skew : few researchers publish many articles and many researchers each publishing only few papers. The shape of the distribution can be well approximated by an inverse power distribution (Power Law) given by the following function :  $f(n) = an^{-k}$  with  $f(n)$  the number of authors having published  $n$  papers,  $a$  and  $k$  being the parameters of the law. When  $k = 2$ , this expression is identical to the

This paper is dedicated to the presentation of an original model of academic competition<sup>5</sup> which encompasses the two stylized facts exposed above. It relies upon the simple following mechanism. We suggest that academic positions differ both in their associate productivity and in the utility they provide to scholars (both due to different wages and other non monetary satisfactions). At the different stages of their career path, agents compete to get the best positions and the (equilibrium) universities hiring and promotion decisions are taken according to the scientific production (or credit) ranking at the previous stage. Therefore, the most productive scientists in their early career are favored in the next stages.

Empirical evidence supports the idea that a mechanism of such kind is at play in the academia. The universities, as well as the research positions they propose, are quite heterogeneous in terms of their associated productivity. Stephan (1996) argues that prestigious institutions are more doted of heavy instrumentation equipment that less established ones cannot afford. Cole (1970) shows that the reputation of the hosting institution generally signals researchers abilities, enabling them both to more efficiently raise funds and to more quickly and widely diffuse their results in the scientific community. Hansen et al. (1978) showed that the quality of the researchers' universities is the critical variable for explaining future production. Cole and Cole (1973) found a university department quality effect on the productivity of physicists. Some empirical evidence also suggests that the attribution of best positions, either through internal promotions or through hiring decisions (Garner, 1979), is mainly based on past scientific production. Zivney and Bertin (1992) showed that the researchers "tenured" in the twenty five most reputed Finance departments of US universities, previously published sensibly more than the average tenured researcher. Having studied the mobility of more than 3,800 economists, Ault, Rutman and Stevenson (1978, 1982) showed that the main determinant of the quality of the first position hosting institution is the quality of the training university (both at "undergraduate" and "graduate" levels) and the quality of the university where the Ph.D. has been defended. Moreover they showed that further "upward mobility" (mobility associated with an increase in the quality of the institution) is mainly explained by past publications (even if the effect is limited). The most productive agents will benefit from better research positions and are in turn likely to publish more. This way the academic competition is dynamically biases in the sense of Merton's cumulative advantage, because the initial successes tend to further improve productivity and in turn favor late successes. Thus, it is in the very nature of the academic employment relationship that lies one of the sources of the accumulative advantage process.

---

one initially proposed by Lotka (1926). Many empirical studies confirmed of the relevance of such distribution for different scientific domains : *e.g.* Murphy (1973) for Humanities, Radhakrishnan and Kernizan (1979) in Computer sciences, Chung and Cox (1990) in Finance, Cox and Chung (1991) in Economics, Newman (2000) for Physics and Medicine, Barabasi et al. (2001) for Maths and Neurosciences, etc.

<sup>5</sup>Several modelling attempts have considered other dimensions of the academic institution. The model introduced in Carmichael (1988) intends to explain the tenure system. Merton and Merton (1989) describe the optimal timing scheme for solving a set of scientific problems. Lazear (1997) models funding agencies. Brock and Durlauf (1999) propose a model of discrete choice of scientific theories when agents have an incentive to conform to the opinion of the community. Lach and Schankerman (2003) model the licensing of scientific discoveries. Carayol and Dalle (2004) propose a model of scientific knowledge accumulation over an increasing set of scientific areas.

More precisely, we model two employers (we refer to universities, but it could be departments or research labs) which propose at each period research positions at all career stages : Ph.D., junior and senior levels to overlapping generations of two researchers. While taking promotion and hiring decisions, universities cannot (or just do not) observe both agents' efforts and cardinal values of past productions : such decisions are taken on the basis of agents' past production ranking<sup>6</sup>. At each stage of the career, positions differ both in terms of their remuneration and their associated productivity. There is a productive premium due to both the accumulated reputation of the host institution and due to positive spillover from ranked-first colleagues of other generations within the university. At the junior and senior stages, the previously most productive agents will select the positions they prefer, while others will accept the remaining academic positions or even choose the outside option (leave the academe). Getting the most productive position improves the chances to win the next competition round that is to get again better productive positions and higher wages. This is the way cumulative advantage in researchers competition is captured in the model. In this paper we also explicitly introduce the competition between universities which also compete at each period to hire the best researchers at junior and senior stages. Therefore the hiring decisions according to agents' ranking, the wages and, the accumulative advantages are endogenously determined in a dynamic setting.

Our main results are the following. We derive researchers equilibrium efforts and show that, as highlighted by several empirical contributions (Zuckerman and Merton, 1972; Allison and Stewart, 1974), the anticipated accumulative advantage improves early career efforts (because it generates dynamic incentives) while it deters late career efforts. Nevertheless, the effect of the accumulative advantage on the whole career scientific production remains ambiguous<sup>7</sup>. As regard the universities competition, we derive markov perfect equilibria of the game under non restrictive assumptions. The equilibrium is stationary in the long run : a fixed cumulative advantage endogenously arises that the leading university confers to its researchers. We precisely compute the equilibrium and the optimal wages and show that the equilibrium wages proposed to ranked-second agents are optimal and that the wages proposed by the leading university to the ranked-first agents may be larger or lower than their optimal counterparts. This is because leading universities do not internalize the positive incentive effect of the wages they propose on scholars hired by other institutions at previous stages.

The paper is organized as follows. The model is presented in the next section. The third section is dedicated to agents' behaviors under such biased competition. The fourth section

---

<sup>6</sup>The structure of the model has much in common with the biased contests literature initially applied to sequential auctioning (Laffont and Tirole, 1988), imperfect measurement of agents' production within firms (Milgrom and Roberts, 1988; Prendergast and Topel, 1996), and lastly career paths within firms that are either autoregressive ("late-beginner effect") for Chiappori et al. (1999) either dynamically correlated ("fast track") for Meyer (1991, 1992).

<sup>7</sup>In a previous contribution (Carayol, 2003) which introduces a more specific model (only one cohort), I specifically studied and found an optimal level of accumulative advantage, i.e. the second stage competitive bias attributed to the first stage winner that optimally balances early career incentive effect and late career disincentive effect.

introduces the universities competition. The fifth section compares optimal to equilibrium wages. The last section concludes.

## 2 The model

### 2.1 Main features

Let us define the academic researchers population as overlapping generations of agents whose career lasts three periods. Let  $p \in \{1, 2, 3\}$  denote the periods at which the agents can be respectively Ph.D., junior researchers, and senior researchers. At each period  $t$  of the discrete time, a fixed cohort  $C^t$  arrives, composed of two researchers. There are two research institutions, say universities  $\{i, j\}$  which hold a position of each stage.

The outcome of scholars work is assumed to result in some aggregated output which can be called research production (that is papers properly weighted for accounting for various quality) or equivalently credit in the peer scientific community. Research production is supposed to be additively separable in efforts over the two first periods of activity. At period  $t$ , the research output of the agent employed in university  $i$  at stage  $p$  of her career is given by :

$$y_i^{p,t} \equiv f^p \left( e_i^{p,t} \right) + b_i^{p,t} + \varepsilon_i^p. \quad (1)$$

It is a function of efforts spent at time  $t$  by the agent being at stage  $p$  of her career  $e_i^{p,t}$ , with  $f^p(\cdot)$  a positive and increasing function which derivative gives the productivity of efforts at the different steps of the career (Ph.D., junior and senior :  $p \in \{1, 2, 3\}$ ).  $f^p(\cdot)$  is assumed to be strictly increasing, concave and null when the agents exerts zero efforts :  $f^{p'} > 0$ ,  $f^{p''} \leq 0$  and  $f^p(0) = 0$ . The term  $\varepsilon_i^p$  is the random specific shock that affects agent  $i$  production at stage  $p$  such as  $E[\varepsilon^p] = 0$ . Let us assume these shocks are *iid* across agents and periods. We define  $\Delta\varepsilon^p$ , as the difference between the individual random shocks at stage  $p$  :  $\Delta\varepsilon^p \equiv \varepsilon_i^p - \varepsilon_j^p$ . The distribution function of this random variable is denoted  $G^p(\cdot)$  and its density function is  $g^p(\cdot)$ . The latter is assumed to be unimodal, continuously differentiable and strictly positive over  $[-\infty, \infty]$  and symmetric around its unique maximum attained at 0 (which implies that  $g^{p'}(0) = 0$ ). The term  $b_i^{p,t}$  gives the surplus of credit which is due to the agents' context of work : it is an attribute of the university in which the agent is working. How this position specific component is formed is stressed as follows :

$$b_i^{p,t} \equiv \alpha_i^t + \beta_i^{p,t} \quad (2)$$

with  $\alpha_i^t$  the effect of the accumulated reputation of the institution on agents' production. For simplicity, we assume that it is independent of the stage considered  $p$ . The vector  $\alpha^t = \left( \alpha_i^t, \alpha_j^t \right)$  synthesizes the reputational advantages of the two universities at  $t$ . The term  $\beta_i^{p,t}$  is the potential production premium due to the previous period ranking of co-workers in the academic institution. It is a positive externality which can be seen as a reputation effect. Alternatively it might also

be thought as a spillover due to non costly interactions (like good advises to next door office colleagues). It can be computed as follows :

$$\beta_i^{p,t} \equiv \beta \times \mathbf{1} \{i | p' | t \text{ ranks first with } p' \neq p\} \quad (3)$$

for  $p, p' = 2, 3$ , with  $\beta$  a positive parameter and  $\mathbf{1} \{\cdot\}$  the indicator function which is equal to 1 if the condition into brackets is verified. The expression “ $i | p' | t$  ranks first” simply means that university  $i$  employs at  $t$  a researcher who is at stage  $p'$  of her carrier and who was ranked-first at the end of the preceding period (period  $t - 1$  and stage  $p' - 1$ ). This assumption formalizes the idea that junior and senior scientists gain some production premium  $\beta$  to be working with well ranked other scientists of the other generation. Ph.D. students (first stage) are assumed to gain equally from the ranks of the two older generations :  $\beta_i^{1,t} = \frac{1}{2} (\beta_i^{2,t} + \beta_i^{3,t})$ . The term  $\alpha_i^t$  is simply constituted of the accumulation of the past premia as follows :

$$\alpha_i^t \equiv \sum_{p=2,3} \sum_{\tau=1 \dots T} \gamma^\tau \beta_i^{p,t-\tau} \quad (4)$$

with some discount factor  $\gamma$  over a given relevant period of time  $T$ . If  $\gamma$  tends to 1, then past and present advantages weight nearly equally in present production. When  $\gamma$  tends to 0, then  $\alpha_i^t$  also tends to zero and the production premium  $b_i^{p,t}$  tends to restrict to the present spillover  $\beta_i^{p,t}$ .

Agents' instantaneous net utility is given by the function  $W(s_i^{p,t}, b_i^{p,t}, e_i^{p,t})$  which is assumed to be additively separable between disutility of efforts and utility as follows :

$$W_i^{p,t} \equiv U(s_i^{p,t}) + \varphi \times \mathbf{1} \{b_i^{p,t} > b_j^{p,t}\} - V(e_i^{p,t}) \quad (5)$$

where  $\varphi > 0$  represents the surplus of satisfaction derived from being in the most prestigious institution. Agents do not value only wages but also the prestige of their host institution<sup>8</sup>.  $U(\cdot)$  is the instantaneous utility function such that agents are assumed to be intra-periods risk averse,  $U(\cdot) : (0, \infty) \rightarrow (-\infty, \infty)$  such as:  $U' > 0; U'' \leq 0$ ,  $\lim_{s \rightarrow 0} U(s) = -\infty$ . The instantaneous disutility of efforts function  $V(\cdot) : (0, \infty) \rightarrow (0, \infty)$  is assumed to be such as :  $V(0) = V'(0) = 0$ ;  $V' > 0$  and  $V'' \geq 0$  over  $\Re^{+*}$  and  $\lim_{e \rightarrow \infty} V'(e) = \infty$ .

The whole career net utility function is assumed to be additively separable between the three periods of the career. We also assume that agents do not have access to the financial market such that they can neither save nor borrow and thus consume their whole revenue received at each period considered. Thus, the total net utility of one agent  $i$  of cohort  $C^t$  actualized at the initial period  $t$  is given by :

$$\overline{W}_i^t = \sum_{p=1,2,3} \delta_a^{p-1} W_i^{p,t+p-1} \quad (6)$$

---

<sup>8</sup>Levin and Stephan (1991) assume that scholars' objective function has a “puzzle solving” argument that is scholars also directly like publishing papers. Our assumption is slightly different in that we rather assume that scholars like being in a distinguished and prestigious institutions.

with  $\delta_a$  the agents' actualization factor.

## 2.2 Information and timing

At each period, universities propose a position at each of the three stages. At the first stage wages are unique and exogenously fixed and agents are assigned to Ph.D. research positions. At the junior and senior stages the universities compete in wages offered. Universities cannot adapt wages with cardinal information on past production : they only compare the scientific production of the two candidates at the previous stage<sup>9</sup>. At both levels, the positions they offer have distinct associated productivity. The universities play simultaneously and have an infinite time horizon. This game is called the Universities Game.

Agents have a life cycle point of view and face an intra-cohort competition. They compete during the first round when they are Ph.D. At the end of the first period, they can access to junior positions proposed by universities. Positions are characterized by an associated utility and a production premium. Given the universities competition, the most productive agent chooses the university that offers the preferred junior position. The other agent accepts the remaining junior position proposed by the other university or defects and takes the outside option. If not, the two agents compete in the following stage. Again, the most productive chooses the senior position he prefers. There is a cumulative advantage because the first winner can choose the junior position that provides an advantage to get the best senior position. At the end of the third period, they retire.

Agents competition is analyzed in the following section while Section 4 deals specifically with universities competition. The time consistency between the two interrelated competitions derives to the fact that efforts are non observable and that cardinal information on production is not available. Previous stage tournaments ranking is the only information available. Therefore, scholars care only about future wages and productive advantages that they consistently believe to be stationary. As we shall show, universities care only about previous period ranking, the other university strategy and the present efforts of its current employees at previous stages when setting their wage proposals.

## 3 Researchers behaviors

We now concentrate on the computation of scientists equilibrium behaviors leaving aside the issue of universities competition that will be treated in the next section. Important for us now, we shall show there that, at each stage, the agent who wins the competition occupies (as expected) at the next period the position that provides the highest satisfaction which also provides the highest productivity. Let the difference in productive advantages  $b^p$  at each stage  $p = 1, 2, 3$  be

---

<sup>9</sup>Thereby, we also assume that universities do not consider the ranking of the Ph.D. stage ranking to hire a senior researcher. This information is either neglected, irrelevant or just lost.

noted  $\Delta b^p$ <sup>10</sup>. We shall also define  $\Pi_i^p$  as the expected utility of the agent employed at university  $i$ <sup>11</sup> from stage  $p$  to the end of the career given its level of information so far. Formally we have

$$\Pi_i^p = \mathbb{E} \sum_{q=p}^3 \delta_a^{q-1} W_i^{q,t+q-1} \quad (7)$$

with  $\mathbb{E}$  the expectation operator.

At each stage, agents maximize expected utility over their remaining career cycle. We use standard backward induction reasoning. Since there is no motive for any competition in the last stage of the career, we have  $e_k^3 = 0, k = i, j$  and third stage production thus restricts to the shock and the potential production premium<sup>12</sup>. We thus concentrate on the two first stages of the career : we first study second stage Nash equilibrium and then first stage subgame perfect Nash equilibrium.

### 3.1 Second stage Nash equilibrium

At the second stage, agent  $i$  chooses her effort level  $e_i^2$ , given  $j$ 's ( $e_j^2$ ) in order to maximize her expected net utility from then. Agents believe that they will get more utility if they win than if they loose such that  $\Delta U^3 > 0$ , a belief which is consistent with universities behaviors as we shall show in the next section. Thus,  $i$ 's program at the beginning of stage 2 consists in

$$\tilde{e}_i^2 \equiv \arg \max \left\{ P(y_i^2 > y_j^2) \times \delta_a (\bar{U}^3 + \varphi) + [1 - P(y_i^2 > y_j^2)] \times \delta_a \underline{U}^3 - V(e_i^2) \right\}. \quad (8)$$

$\bar{U}^3$  is the utility derived from wage if the second contest is won and  $\underline{U}^3$  if it is lost. The Nash equilibrium effort level ( $\tilde{e}_i^2$ ) maximize the expected net utility actualized at the second career stage. It is equal to the probability of winning the second tournament times the utility he will receive if he wins the tournament, plus the probability to loose that tournament times the utility received if he looses it, net of the disutility of efforts<sup>13</sup>. An identical program could be written for  $j$ . The simultaneous resolution of the two programs detailed in the Appendix A leads to a unique Nash equilibrium which is symmetric and given by

$$\tilde{e}^2 = \Theta_2^{-1} (\delta_a (\Delta U^3 + \varphi) g^2 (\Delta b^2)), \quad (9)$$

with function  $\Theta_2(\cdot)$  defined by  $\Theta_2(x) = V'(x)/f^{2'}(x)$ . This function is null at 0 and strictly increasing from then (and thus so is its inverse function  $\Theta_2^{-1}(\cdot)$ ). Therefore, second period efforts are increasing with agents' actualization factor, and the differences in third stage differences in utility whatever they come from difference in wages ( $\Delta U^3$ ) or in satisfaction derived from being in a prestigious institution ( $\varphi$ ). The accumulative advantage ( $\Delta b^2$ ) effects are negative (for details see the Appendix).

<sup>10</sup>Since the analysis in this section will be limited to only one cohort behavior, we will remove time superscript and uniquely refer to career periods (given by  $p$ ).

<sup>11</sup>In this section "agent  $i$ " and "the agent employed by university  $i$ " have the same meaning.

<sup>12</sup>Clearly senior researchers do exert efforts in real life. Nevertheless, this behavior seems not to respond to the kind of motives that are at stake in this model.

<sup>13</sup>We omit second stage utility since it is independant of second stage efforts.

### 3.2 First stage subgame perfect Nash equilibrium

We now turn to the first period agents' behaviors. Agent  $i$  objective is to determine her first period effort level in order to maximize expected net utility, that is

$$\tilde{e}_i^1 = \arg \max \{ \Pi_i^1 \}. \quad (10)$$

This maximization program is different from the second period one since the first period success influences the second stage competition. The unique and symmetric subgame perfect Nash equilibrium (again detailed computations are in the Appendix A) is given by :

$$\tilde{e}^1 = \Theta_1^{-1} (g^1 (\Delta b^1) \delta_a [\Delta U^2 + \varphi + \delta_a (2G^2 (\Delta b^2) - 1) (\Delta U^3 + \varphi)]) \quad (11)$$

with  $\Theta_1(\cdot)$  such that  $\Theta_1(x) = V'(x)/f^{1'}(x)$  and  $\Theta_1^{-1}$  is increasing. Moreover, knowing that  $\Delta b^2 > 0$  and that  $g^2(\cdot)$  is symmetric around its unique extremum at 0, then  $G^2(\Delta b^2) \geq 1/2$  and thus  $2G^2(\Delta b^2) - 1 \geq 0$ . Thus first stage equilibrium efforts  $\tilde{e}^1$  are increasing with the actualization factor and with the differences in utilities at the junior and the senior stages ( $\Delta U^2$  and  $\Delta U^3$ ) whatever they come from the differences in wages or from the differences in satisfaction to be in the most prestigious institution. The accumulative advantage ( $\Delta b^2$ ) effects are positive while the effects of the initial advantage ( $\Delta b^1$ ) are negative (for details see the Appendix).

These results are summed up in Proposition 1 below. It confirms the results of several empirical according to which the accumulative advantage stimulates early career efforts while it diminishes late career efforts.

**Proposition 1.** *Agents' equilibrium efforts are unique and symmetric at each stage of the career cycle. Equilibrium efforts decrease with the differences in utility provided by positions at the remaining stages of the career. The accumulative advantage which favors in the second stage the winner of the first competition increases first period equilibrium efforts while it decreases the second period efforts. The first stage advantage decreases first period efforts.*

## 4 Universities competition

In this section, we shall focus on the Universities Game. At the end of each period, the scholars' ranking is public knowledge. At the beginning of the next period, the universities propose one position at each stage  $p = 1, 2, 3$ . To fulfill their available junior and senior positions (agents are assigned exogenously to Ph.D. positions at the beginning of the career), universities compete to hire the best researchers. Neither the universities care about the institutions where the researchers were previously employed, nor relevant cardinal information on past production is available (or relevant as regard the institutional constraints). The universities compete simultaneously in wages. The competition is asymmetric with respect to their respective reputations. Universities consider the game as lasting infinitely.

Let us denote by  $\Omega_i$  the objective function of university  $i$  at some time period  $t_0$ . Without loss of generality, we will consider  $t_0 = 0$  in order to avoid cumbersome writings. It is equal to the discounted net surplus captured by university  $i$  over an infinite period of time as follows,

$$\Omega_i = \sum_{\tau=0}^{\infty} \sum_{p=1}^3 \delta^{\tau} (\psi y_i^{p,\tau} - s_i^{p,\tau}). \quad (12)$$

The parameter  $\psi > 0$  gives the unitary value of scientific knowledge captured or just considered<sup>14</sup> by the university. It is assumed to be homogeneous or normalized. Parameter  $\delta$  is the discount factor.

Let now  $\Delta y_i^{p,t}$  denote the net surplus of production university  $i$  gets from employing at stage  $p$  and period  $t$  the scientist who won her preceding academic tournament (as compared to hiring the one who did not). It directly derives from the preceding section that this surplus is only composed of the direct productive complement a ranked-first agent provides to other researchers employed by  $i$  (at other stages). It is independent of  $i$  and can be computed as follows :

$$\Delta y_i^{p,t} = \Delta y^p = \frac{3}{2}\beta, \quad (13)$$

which is directly computed from agents' production function equation (1) and from the equation which sets the direct externality (3). We denote by  $\underline{y}^{p,t}$  the expected production only due an agent's efforts (without considering the premium). It is only affected by her equilibrium efforts (which are unique at each stage as shown in Section 3).

The universities payoffs at period  $t$  can be written as a function of the wages proposed by the two universities at the two last stages summed up in vector  $\mathbf{s}^t = \left( s_k^{p,t} \right)_{k=i,j;p=2,3}$ . It is given by :

$$\pi_i(\mathbf{s}^t) = \sum_{p=1}^3 \left( \psi \underline{y}^{p,t} - s_i^{p,t} \right) \times \mathbf{1} \left\{ \Pi_i^p \geq \widehat{W}^p \right\} + \psi \sum_{p=2}^3 \frac{3}{2}\beta \times \mathbf{1} \left\{ \Pi_i^p > \Pi_j^p \text{ and } \Pi_i^p \geq \widehat{W}^p \right\}. \quad (14)$$

University  $i$  must offer agents a wage that provides a higher expected utility than their reserve utility level outside the university system ( $\Pi_i^p < \widehat{W}$ ). Otherwise, agents always defect and the university gets a null payoff. The second component of the right hand side of the equation indicates that each stage  $p = 2, 3$ , if university  $i$  provides the highest expected utility given the level of information of agents so far ( $\Pi_i^p > \Pi_j^p$ ), it hires the researcher ranked-first and captures the surplus of production as given in (13). Otherwise, the university hires the other researcher and can not capture the surplus in revenues associated with the production premia given in (13).

At each period, an action of university  $k = i, j$  in the Universities Game is a vector  $\mathbf{s}_k^t = \left( s_k^{2,t}, s_k^{3,t} \right)$  of the two wages proposed at  $t$ . The history of the game so far noted  $h^t \in H^t$  ( $H^t$

---

<sup>14</sup>If the university is controlled by any external institution or body having its own goals (e.g. a state), the rate  $\psi$  considered in the objective function might be higher than the effective rate of returns of scientific knowledge on the university budget. It would become closer to its social value. The comparison between  $\psi$  and the real social value of scientific knowledge comes in Section 5.

is the set of all possible histories in period  $t$ ) is the collection of all previous actions such that  $h^t = h^{t-1} \cup (\mathbf{s}^{t-1})$ .  $H$  is the set of all possible histories over time such that  $H = \cup_{t=0}^{\infty} H^t$ . A pure behavioral strategy at  $t$  is a function  $\rho_i^t : H^t \times \mathfrak{R}^{+2} \rightarrow \mathfrak{R}^{+2}$ , which gives a couple of wages at  $t$  (an action  $\mathbf{s}_i^t$ ) from each possible history at  $t$  and each possible wage simultaneously proposed by the other university ( $\mathbf{s}_j^t$ ).

We are interested in Markov Perfect Equilibria (MPE) (see Maskin and Tirole, 2001) of the Universities game. So we restrict to markov strategies, that is strategies that are not function of the whole history of the game but only of the state of the system. The vector  $\boldsymbol{\alpha}^t \in \mathfrak{R}^{+2}$  which synthesizes the reputational premia of the two universities at  $t$ , is the payoffs relevant state of the system because no other past variable does consistently influence present actions. We consider only strategies of the form :  $\sigma_i^t : \mathfrak{R}^{+2} \times \mathfrak{R}^{+2} \rightarrow \mathfrak{R}^{+2}$ . Such strategies give, for each action of the opponent and for each given reputation levels, a couple of wages  $\mathbf{s}_i^t$ . It is of the form :  $\sigma_i^t = \sigma_i^t(\mathbf{s}_j^t, \boldsymbol{\alpha}^t)$ . A MPE is a couple of strategies  $\hat{\boldsymbol{\sigma}}^t = (\hat{\boldsymbol{\sigma}}_i^t, \hat{\boldsymbol{\sigma}}_j^t)$  such that  $\hat{\boldsymbol{\sigma}}_i^t$  and  $\hat{\boldsymbol{\sigma}}_j^t$  are best responses to each other and are markov strategies.

The MPE notion relies much on the idea that small causes have minor effects. In the dynamic programming spirit, agents are assumed to simultaneously maximize their continuation equations which in the context of the Universities Game can be set as follows :

$$\theta_i(\mathbf{s}^t, \boldsymbol{\alpha}^t) = \sum_{p=1}^3 (\psi \underline{y}^{p,t} - s_i^{p,t}) \times \mathbf{1} \left\{ \Pi_i^p \geq \widehat{W}^p \right\} + \sum_{p=2}^3 B \times \mathbf{1} \left\{ \Pi_i^p > \Pi_j^p \text{ and } \Pi_i^p \geq \widehat{W}^p \right\}. \quad (15)$$

The expression is similar than the payoff function (14) with the exception that now, hiring the ranked-first scientist at the junior and senior levels brings some delayed productive surplus due to the increase of reputation its causes to the host university. The increment of reputation depreciates over years at rate  $\gamma$  and is discounted by factor  $\delta$  over an infinite horizon. The payoff surplus is thus  $\frac{3}{2}\beta\psi \sum_{t=1}^{\infty} \delta^t \gamma^t$ . When one adds to this term the direct spillovers  $\frac{3}{2}\beta\psi$  already present in (14), it becomes equal to  $B \equiv \frac{3}{2}\beta\psi \sum_{t=0}^{\infty} \delta^t \gamma^t = \frac{3}{2}\beta\psi (1 - \delta\gamma)^{-1}$ .

The simultaneous maximization, at each period, of the two universities continuation equations leads to the MPE. The intuitions for the equilibria are the following. By convention, and without loss of generality, let  $i$  be the leading university at the period considered  $t$  while  $j$  is the other university, that is :  $\alpha_i^t > \alpha_j^t$ . University  $i$  can attract the best scientist with a lower salary at stages 2 and 3 because agents value being in the most reputed university and, if they are about to enter the junior stage of the career, they also know that working in this university will increase their probability to win the next tournament. Since both institutions value equally recruiting the best researcher, such asymmetry provides university  $i$  a decisive advantage. Indeed, university  $i$  can propose a wage such that university  $j$  can not set a wage that might attract a ranked-first researcher without having a lower return than just hiring the ranked-second agent. The best rate at which university  $j$  can attract the ranked-second agent is by setting the minimal wages which saturate agents participation constraints. Theorem 2 below states this more rigorously.

**Theorem 2.** *Given Assumptions 3, at any period  $t > 0$  of the Universities Game, the MPE  $\widehat{\sigma}^t = (\widehat{\sigma}_i^t, \widehat{\sigma}_j^t)$  is such that, if  $\alpha_i^t > \alpha_j^t$ , the equilibrium wages  $\widehat{\mathbf{s}}^t = (\widehat{\mathbf{s}}_i^t, \widehat{\mathbf{s}}_j^t)$  are  $\widehat{\mathbf{s}}_i^t = (\overline{s}^{p,t})_{p=2,3}$  and  $\widehat{\mathbf{s}}_j^t = (\underline{s}^{p,t})_{p=2,3}$  such that i)  $\Pi_j^p = \widehat{W}^p$  for all  $p = 2, 3$ , and ii)  $\widehat{\mathbf{s}}_i^t = \arg \max_{s_i^{p,t}, p=2,3} \sum_{p=1}^3 (\psi y^{p,t} - s_i^{p,t})$  subject to the incentive constraints (9 and 11), subject to the participation constraints and subject to competition constraints that ensure  $i$  attracts the ranked-first agents. The competition constraints are given by the following condition. For any vector of wages  $\mathbf{s}^t$  that differs from  $\widehat{\mathbf{s}}^t$  only in wages proposed by  $j$  (possibly the two)  $s_j^{p,t} = s^{p'} \neq \underline{s}^{p,t}$  such that  $\Pi_j^p \geq \Pi_i^p$ , then  $\theta_j(\mathbf{s}^t, \boldsymbol{\alpha}^t) < \theta_j(\widehat{\mathbf{s}}^t, \boldsymbol{\alpha}^t)$  (any move allowing  $j$  to hire ranked-first agent(s) would be detrimental to  $j$ ).*

**Proof.** The proof is in the Appendix A.

**Assumptions 3.** *A3.1) for all vector of wages  $\mathbf{s}'''^t$  identical to  $\mathbf{s}^t$  except that at any given stage (possibly the two),  $s_j^{p,t}$  is equal to 0 instead of  $\underline{s}^{p,t}$ , we have  $\theta_i(\mathbf{s}'''^t, \boldsymbol{\alpha}^t) \leq \theta_i(\mathbf{s}^t, \boldsymbol{\alpha}^t)$ ; A3.2) for all vector of wages  $\mathbf{s}'''^t$  identical to  $\mathbf{s}^t$  except that at any given stage  $p$  (possibly the two),  $s_i^{p,t}$  is equal to  $\underline{s}^{p,t}$  instead of  $\overline{s}^{p,t}$ , we have  $\theta_i(\mathbf{s}'''^t, \boldsymbol{\alpha}^t) \leq \theta_i(\mathbf{s}^t, \boldsymbol{\alpha}^t)$ .*

This assumption simply rules out trivial and uninteresting configurations. It states that, at the equilibrium and at both stages, the university which has not the reputational advantage prefers to hire the ranked-second worker rather than hiring no-one (A3.1) and; the university which has the reputational advantage prefers to hire the ranked-first worker at the equilibrium wage at both stages rather than hiring the ranked-second worker at the wage which saturates agent outside option utility (A3.2).

Theorem 2 shows how the other university rivalry plays as a *competition constraint* on the leading university. The *competition equilibrium* is said to arise when the leading university saturates the competition constraints. In this configuration the leading university sets its third stage wage  $\overline{s}^{3,t}$  such that

$$U(\underline{s}^{3,t} + B) = U(\overline{s}^{3,t}) + \varphi, \quad (16)$$

that is, the ranked-first agent, at the beginning of the senior stage, would have an equal satisfaction being hired by the less reputed university which would offer (in addition to the wage that saturates the participation constraint  $\underline{s}^{3,t}$ ) to the agent all the value of the productive (direct and delayed) premium this agent would bring in<sup>15</sup> or being hired in the leading university.

The leading university sets its second stage wage  $\overline{s}^2$  such that

$$U(\underline{s}^{2,t} + B) = U(\overline{s}^{2,t}) + \varphi + \delta_a [2G(b) - 1] (U(\overline{s}^{3,t}) + \varphi - U(\underline{s}^{3,t})). \quad (17)$$

At the beginning of the junior stage, the ranked-first agent would have again an equal expected utility being hired by the less reputed university which would offer the value of its productive premium ( $B$ ) or being hired by the leading university and benefiting from both the direct

---

<sup>15</sup>  $B$  is the maximal amount the non leading university is ready to offer to the ranked-first agent on the top of the wage that it would give to the ranked second agent.

satisfaction working there ( $\varphi$ ) and the discounted surplus of expected utility due to the increased chance to win the next contest.

Nevertheless the incentive constraint is still effective because their employees consistently anticipate that the proposed wages are constant in time. The leading university knows that the second period wage proposed has an incentive effect on the agent who is currently employed by the university at Ph.D. stage. Similarly, the leading university knows that the senior wage proposed has an incentive effect on both the Ph.D. and the junior who are employed in this university. Thus there is no reason to exclude *a priori* a configuration in which the competition constraint is not saturated due to purely incentive purposes. The leading university wages might differ depending whether the competition constraints are saturated or not.

Let us define  $(\bar{s}^{p,t*})_{p=2,3}$  the full incentive maximizing wages of the leading university  $i$ , that are solution of the optimization program of Theorem 2 but now irrespective to the competition condition. If  $\bar{s}^{p,t*} > \bar{s}^{p,t}, \forall p = 2, 3$  then the competition constraint is not effective and the *full incentive maximizing equilibrium* is said to arise (none of the incentive constraints is saturated). If  $\bar{s}^{2,t*} < \bar{s}^{2,t}$  and  $\bar{s}^{3,t*} > \bar{s}^{3,t}$ , the *mixed equilibrium* arises. The incentive effect is then prevalent only for the senior wage<sup>16</sup> while the junior wage offered by the leading university saturates the competition constraints.

**Corollary 4.** *Theorem 2 leads to three possible Markov Perfect Equilibria which differ only in the wages proposed by the leading university depending on whether the competition constraints are saturated : i) the competition equilibrium if  $\bar{s}^{p,t} > \bar{s}^{p,t*}, \forall p = 2, 3$ ; ii) the mixed incentive equilibrium, if  $\bar{s}^{2,t*} < \bar{s}^{2,t}$  and  $\bar{s}^{3,t*} > \bar{s}^{3,t}$  and, iii) the full incentive maximizing equilibrium if  $\bar{s}^{p,t*} > \bar{s}^{p,t}, \forall p = 2, 3$ .*

Now we investigate the long run implications of Theorem 2 which shows that the leading university thus always attracts the ranked-first scholars. There is thus a path dependent process since the equilibrium of the university competition game preserves the competitive advantage of the leading university which tends to some fixed value as stated in Corollary 5 below.

**Corollary 5.** *The MPE of the Universities Game preserves the competitive advantage of the leading university over time : the most reputed university hires the winning agents at junior and senior stages and thus conserves full advantage over time. In the long run ( $t \rightarrow \infty$ ), the endogenous competition advantages the leading university confers to its employees tend to  $b \equiv \beta(1 + \gamma) / (1 - \gamma)$  for an infinite reputation relevant period  $T$ .*

**Proof.** According to Theorem 2, the leading institution hires at both stages the ranked-first agents and thus preserves its advantage for ever. When  $t$  tend to infinity, any productive premia the non leading university might had (by any kind of accident) tends to zero (due to the discount). Then, the productive advantage ( $\Delta b^{p,t}$ ) of any agent employed by the leading university is strictly equal to the productive premium ( $b^{p,t}$ ). In the long run, when  $t \rightarrow \infty$ ,

---

<sup>16</sup>Senior wages have a higher incentive effect because they play positively on both the Ph.D. and the junior stages efforts while the junior wages only impact on agents holding a Ph.D. position.

it also becomes invariant in time and is computed as follows :  $b^{p,t} = \beta + \sum_{p=2,3} \sum_{\tau=1,\dots,T} \gamma^\tau \beta$  which tends to  $b \equiv \beta(1 + \gamma) / (1 - \gamma)$  when the relevant period  $T$  tends to infinity.  $\square$

## 4 Welfare analysis

This section is dedicated to the welfare analysis. Optimal wages are computed in the first subsection under some simple specifications of the utility, the disutility and the production functions. In the second subsection, we compute the equilibrium wages given these specifications and study how they relate to optimal ones.

### 4.1 Optimal wages

We assume that the social surplus created by the academic activity is simply obtained, through function  $\Phi(\cdot)$ , by the actualized sum of the individual productions time a given parameter  $\phi > 0$  which gives the unitary social value of scientific knowledge which is assumed to be homogeneous (or normalized)<sup>17</sup>. Thus, the total surplus actualized at period  $t_0 = 0$  generated is given by :

$$\Phi \equiv \sum_{\tau=0}^{\infty} \sum_{k=i,j} \sum_{p=1}^3 \delta^\tau (\phi y_k^{p,\tau} - s_k^{p,\tau}) \quad (18)$$

with  $\delta$ , the social actualization factor.

The program of the central planner is to set the optimal recruitment scheme at each period and stage. We assume that the central planner has exactly the same level of information as universities : at each period, it can only use ordinal information on the previous period ranking<sup>18</sup>. The planner naturally offers the best positions to the ranked-first agent (as universities competition does) in order to fully preserve the continuation career incentives. It sets the optimal wages as follow  $\widehat{\mathbf{S}}^t = (\widehat{s}^2, \widehat{s}^2, \widehat{s}^3, \widehat{s}^3) = \arg \max_{\mathbf{S}^t} \Phi(\mathbf{S}^t)$  under the incentive constraints given in (8) and (10) and the participation constraints. By convention, wages  $\widehat{s}^p$  are proposed to the ranked-first agents and wages  $\bar{s}^p$  are proposed to ranked-second ones.

We now specify the functions  $U(\cdot)$ ,  $V(\cdot)$  and  $f^p(\cdot)$  according to their properties given in Section 2. The utility function is simply assumed to be  $U(s) = \ln s$ . The disutility function is assumed to be quadratic in efforts :  $V(e) = \frac{1}{2}ce^2$ . The production functions of scientific knowledge are assumed to be linear in efforts, with :  $f^2(e) = \mu f^1(e) = \mu ae$ . The strictly positive parameter  $\mu$  gives the increase in agents' productivity between the two first period of their career. If we have  $\mu > 1$ , then agents' productivity increases through the career path. Let also the  $\Delta\epsilon^d$  be identically distributed across the different periods of the career, that is  $g^d(\cdot) = g(\cdot), \forall d$ .

---

<sup>17</sup>Notice we do not assume that the social value of knowledge  $\phi$  and the value considered by the universities  $\psi$  are identical.

<sup>18</sup>Therefore, our analysis can be seen as a second-best approach as regard an omniscient social planner.

We focus on the long run wages ( $t \rightarrow \infty$ ), for which we know that  $\Delta b^{p,t} \rightarrow b, \forall p = 1, 2, 3$ . Again, the long term wages are consistently expected to be stationary by agents. They anticipate that the next period wages will be the same they observe of the current period. Given such anticipations, the central planner has no reason to modify the long run wages in time.

The social planner sets the lowest optimal wages at junior and senior stages so as to saturate agents' participation constraints. Thus, the wages of the ranked-second agents at both stages are :  $\bar{s}^2 = \exp(\widehat{W}^2)$ ,<sup>19</sup> and  $\bar{s}^3 = \exp(\widehat{W}^3)$ . The wages proposed to the ranked-first agents are simply derived from the two FOCs of the social planner program settled with regard to  $\hat{s}^2$ , and  $\hat{s}^3$  (detailed computations are in the Appendix). It comes :

$$\hat{s}^2 = 2\phi \frac{a^2}{c} \delta_a g(b), \quad (19)$$

$$\hat{s}^3 = 2\phi \frac{a^2}{c} \delta_a g(b) (\delta_a (2G(b) - 1) + \mu^2). \quad (20)$$

The optimal ranked-first junior wages decrease with the accumulative advantage  $b$ . The effect of  $b$  on  $\hat{s}^3$  is ambiguous. The optimal wages also increase with the productivity of agents' efforts ( $a$ ), the unitary social value of scientific knowledge ( $\phi$ ) and agents' actualization factor ( $\delta_a$ ). They decrease with the unitary cost of efforts ( $c$ ).

## 4.2 Comparing optimal and equilibrium wages

Let us compute the equilibrium wages given the specifications introduced in the previous subsection. As stated in *i*) of Theorem 2, the lowest equilibrium wages saturate the participation constraints then :  $\underline{s}^2 = \exp(\widehat{W}^2)$ ,  $\underline{s}^3 = \exp(\widehat{W}^3)$ . These wages are strictly identical to their optimal counterparts. Let us now focus on the *competition equilibrium*. The equilibrium wages of the leading university are simply obtained using equations (16) and (17), introducing the specifications and after few some combinations. It comes :

$$\bar{s}^2 = \exp \left\{ \ln \left( e^{\widehat{W}^3} + B \right) [1 - \delta_a (2G(b) - 1)] - \varphi - \delta_a (2G(b) - 1) \left( \varphi - \widehat{W}^3 \right) \right\} \quad (21)$$

$$\bar{s}^3 = \left( e^{\widehat{W}^3} + B \right) e^{-\varphi} \quad (22)$$

These wages saturate the constraint that the leading university attracts the two ranked-first agents for any profitable offer of the opponent university. The other university is ready to offer (in wage) to a ranked-first researcher up to the totality of the spillovers and reputation premia he would bring in. The ranked-first researcher accepts the position if such wage provides him a higher utility than the satisfaction to be in the most reputed university and (at the junior stage only) the surplus of expected utility due to the higher probability to win the next tournament. Thus the leading university proposes a wage to the ranked-first agent so that the wages the

---

<sup>19</sup>We assume here, so as to simplify the notations, that agents compare the second stage outside option with the current period utility (and not with the whole career expected utility flow). In short, agents compare  $\widehat{W}^2$  with  $\underline{U}^2$  instead of  $\Pi^2$ .

other university should propose to attract her are sufficiently high so that it prefers to hire the ranked-second at the best rate.

If the leading university prefers to pay more its employees just because it increases its payoffs thanks to the surplus of incentives it brings to its current employees at the previous stages we are in the *full incentive maximizing equilibrium*. It can easily be shown that then the wages proposed by the leading university (set unrespective to the competition constraints), can be simply derived from the optimal wages as follows

$$\bar{s}^{p*} = \frac{\psi}{2\phi} \hat{s}^p, \forall p = 2, 3. \quad (23)$$

Universities value scientific knowledge production at rate  $\psi$  instead of  $\phi$ , and they only take into account the incentive effect of wages on their own employees. If  $\frac{\psi}{2\phi} \hat{s}^p > \bar{s}^p, \forall p = 2, 3$  and if the university associated returns are higher then the university set the proposed wage so as to maximize the incentives provided to its currently employed Ph.D. agents. Notice that the equilibrium wages will be lower than their optimal counterparts only if  $\psi > 2\phi$ , that is if the universities value knowledge at least twice its social value<sup>20</sup>.

**Proposition 6.** *Optimal and equilibrium wages of the ranked-second agents at junior and senior career stages are equally set so as to saturate agents participation constraints. The competition and full incentive equilibria wages offered to the ranked-first agents can be either greater or lower than the optimal ones depending on the values of the parameters.*

**Proof.** The proof is trivial from the comparison of equations (21) with (19), and (22) with (20) and considering equation (23) for the full incentive maximizing equilibrium.

## 5 Conclusion

In this paper we have introduced a model of academic competition which intends to capture both the life-cycle effect and the cumulative advantages effect of the academic reputation based reward system on the provision of individual incentives. We have suggested a mechanism according to which such effect is rooted in the employment relationship. Research positions are intrinsically unequally productive and the attribution of the best ones is based on the ranking on past scientific production. Unequal productivity is essentially due to some positive externality best ranked agents have on the scholars employed by their university and to a positive effect of the accumulated reputation of the employing university which is endogenously determined by the past successes of the universities in the recruitment of the most reputed scholars.

Our results highlight that the accumulative advantage has negative effects on incentives at all but the first stage of the career at which the effect is ambiguous. The most important results of this paper concern the other side of the coin, namely the between-universities competition. The leading university always hires the ranked-first agents at junior and senior stages at the

---

<sup>20</sup>for space onstraints, we do not examine the mixed equilibrium here the analysis of which does not bring much.

equilibrium. The accumulative advantage the leading university confers to its employees is endogenously generated in the long run and is stationary. The wages proposed by the non leading university are optimal. There are three possible equilibrium wages proposed by the leading university. In the competition equilibrium, the leading university sets the wages so as to saturate the competition constraint which ensure it hires the ranked-first scholars. In the full incentive equilibrium, the leading university sets wage proposals so as to maximize the incentives provided to the agents it currently employs at previous stages. In the mixed equilibrium, only the junior wage saturates the competition constraint.

In all cases, there is no reason why the equilibrium wages offered to ranked-first agents should correspond to the optimal ones. In the competition equilibrium, the leading university cares only about the capacity of the opponent university to attract ranked-first agents. If the competition between universities is very low, the full incentive equilibrium is likely to arise. Then, the leading university may not consider the competition constraint and rather focus on the provision of incentives to agents (as in the optimal configuration) but in a very specific and partial manner. The leading university does not take into account the incentives its wages have on the other university employees. More : since universities cannot control neither efforts nor production at each stage, no incentive can be provided to their seniors and junior wages impact only on the Ph.D. they hire. It is only if the leading university values knowledge twice more than its social value (which is rather unlikely) that the full incentive equilibrium wages of the leading university are higher than their optimal counterparts.

## Appendix A

### Computation of the second stage Nash equilibrium

The first order condition of program (8) is

$$\delta_a (\Delta U^3 + \varphi) \times \partial P (y_i^2 > y_j^2) / \partial e_i^2 = \partial V (e_i^2) / \partial e_i^2. \quad (A1)$$

Notice that the probability that  $i$  wins the second tournament is given by

$$P (y_i^2 > y_j^2) = P (f^2 (e_i^2) + \Delta b^2 + \Delta \varepsilon^2 > f^2 (e_j^2)) = [1 - G^2 (f^2 (e_j^2) - f^2 (e_i^2) - \Delta b^2)].$$

When differentiating that expression with respect to  $i$ 's efforts, one obtains :

$$\partial P (y_i^2 > y_j^2) / \partial e_i^2 = f^{2'} (e_i^2) \times g^2 (f^2 (e_j^2) - f^2 (e_i^2) - \Delta b^2).$$

Introducing this expressions in the first order condition (A1), one gets :

$$f^{2'} (e_i^2) \times g^2 (f^2 (e_j^2) - f^2 (e_i^2) - \Delta b^2) \delta_a (\Delta U^3 + \varphi) = V' (e_i^2)$$

Let us define the function  $\Theta_2 (\cdot)$ , such as  $\Theta_2 (x) = V' (x) / f^{2'} (x)$ . This function is defined over  $\mathfrak{R}^+$ ,  $\Theta^2 (\cdot) : (0, \infty) \rightarrow (0, \infty)$ . Since  $V' (0) = 0$ , this function is null at 0 ( $\Theta (0) = 0$ ). Moreover, having  $V' > 0, V'' > 0, f^{2'} > 0, f^{2''} \leq 0$ , it can easily be shown that this function is strictly

increasing :  $\Theta_2' > 0$ . Thus, its inverse function  $\Theta_2^{-1}(\cdot) : (0, \infty) \rightarrow (0, \infty)$  is also increasing. Then, one can rewrite the two first order conditions using these new notations :

$$\Theta_2(e_i^2) = \delta_a (\Delta U^3 + \varphi) g^2 (f^2(e_j^2) - f^2(e_i^2) - \Delta b^2),$$

$$\Theta_2(e_j^2) = \delta_a (\Delta U^3 + \varphi) g^2 (f^2(e_i^2) - f^2(e_j^2) - \Delta b^2).$$

Given the assumptions formulated so far, these two equations are of the form  $e_i^2 = h(e_j^2)$  and  $e_j^2 = h(e_i^2)$  with  $h(\cdot)$  a continuous function over  $\mathfrak{R}^+$ . Therefore, if an equilibrium exists, it is necessarily symmetric of the form  $e_i^2 = e_j^2 = e^2$  such that  $e^2 = h(h(e^2))$ . This equilibrium then satisfies the following expression :

$$\Theta_2(e^2) = \delta_a (\Delta U^3 + \varphi) g^2 (\Delta b^2).$$

Knowing that that  $\Theta_2(\cdot)$  is strictly positive, is null at zero, is strictly increasing and that  $\lim_{e \rightarrow \infty} \Theta_2(e) = \infty$  (since by assumption  $\lim_{e \rightarrow \infty} V'(e) = \infty$  and  $f^{2''} \leq 0$ ), this equation admits a unique solution. Moreover, since  $g^2 > 0$ ,  $\Delta U^3 > 0$  and  $f^{2'} > 0$ , this solution is strictly positive. The unique symmetric second stage Nash equilibrium is thus given by

$$\tilde{e}^2 = \Theta_2^{-1}(\delta_a (\Delta U^3 + \varphi) g^2 (\Delta b^2)) \quad \square.$$

### Computation of the first stage subgame Perfect Nash equilibrium

Let  $p_i$  denote the probability that, if the agent employed by university  $i$  has won the first stage competition, he will also win the second stage one. Since the second tournament is influenced by the results of the first one,  $p_i$  is a conditional probability. Since the second stage equilibrium efforts are identical (given in 9), that conditional probability is independent of agents' identity ( $p = p_i = p_j$ ). It can be computed as follows :

$$p = P(e^2 + \varepsilon_i + \Delta b^2 > e^2 + \varepsilon_j) = 1 - P(\Delta \varepsilon < -\Delta b^2) = 1 - G^2(-\Delta b^2).$$

Referring to the assumption that  $g(\cdot)$  is symmetric around 0, one can write :  $p = G^2(\Delta b^2)$ .

We note  $\Delta \Pi^p$  the surplus of expected utility received from stage  $p$  (included) over the remaining career cycle it provides to given that stage  $p$  is won and the expected utility given it is lost. Formally it is given by

$$\Delta \Pi^p = \Pi_i^p |_{y_i^p > y_j^p} - \Pi_i^p |_{y_i^p < y_j^p},$$

with  $\Pi_i^p |_{y_i^p > y_j^p}$  the expected utility of agent  $i$  conditioned on  $i$  will win the ongoing  $p$ 's period tournament.

Using these notations and definitions and after several recombinations, we can rewrite  $\Delta \Pi^1$  as follows :

$$\Delta \Pi^1 = \delta_a (\bar{U}^2 + \varphi) + \delta_a^2 \left[ p (\bar{U}^3 + \varphi) + (1-p) \underline{U}^3 \right] - \delta_a \underline{U}^2 - \delta_a^2 \left[ (1-p) (\bar{U}^3 + \varphi) + p \underline{U}^3 \right].$$

After a few simplifications, it comes :

$$\Delta\Pi^1 = \delta_a (\Delta U^2 + \varphi) + \delta_a^2 (2G^2 (\Delta b^2) - 1) (\Delta U^3 + \varphi),$$

with  $\Delta U^2$  the difference in utility between having won the first tournament and having lost it. Introducing that expression in the first order condition of the first period maximization program (10), it comes<sup>21</sup> :

$$\Theta_1^{-1} (e^1) = g^1 (\Delta b^1) \delta_a [\Delta U^2 + \delta_a (2G^2 (\Delta b^2) - 1) \Delta U^3],$$

with  $\Theta_1 (\cdot)$  defined similarly than with  $\Theta_2 (\cdot)$ , that is  $\Theta_1 (x) = V'(x) / f^{1'}(x)$ . The equilibrium is symmetric and unique for the same reason than for the second period. The final expression of the equilibrium efforts (11) follows  $\square$ .

### The incentive properties of the accumulative advantage

We here specifically study the effects of the competitive advantages at the first two stages of the career ( $\Delta b^1$  and  $\Delta b^2$ ) on the equilibrium efforts ( $\tilde{e}^1$  and  $\tilde{e}^2$ ).

The second period efforts are independent of the first period advantage. In order to characterize how the cumulative advantage  $\Delta b^2$  affects the second period equilibrium efforts, we differentiate both sides of equation (11) with respect to  $\Delta b^2$  :

$$\partial \tilde{e}^2 / \partial \Delta b^2 = g^{2'} (\Delta b^2) \delta_a (\Delta U^3 + \varphi) \times \Theta_1^{-1'} (g^2 (\Delta b^2) \delta_a (\Delta U^3 + \varphi)) \quad (\leq 0)$$

We know that  $\Theta_1^{-1}$  is an increasing function. Moreover, knowing that  $\Delta b^2 > 0$  and that  $g^2 (\cdot)$  has its unique extremum at 0, thus  $g^{2'} (\Delta b^2)$  is strictly negative. Thus, we can conclude that  $\partial \tilde{e}^2 / \partial \Delta b^2 \leq 0$ .

The first period equilibrium efforts are function of both the first and second stages accumulative advantages. From equation (9), we obviously have  $\partial \tilde{e}^1 / \partial \Delta b^1 < 0$ , since  $\Theta_1^{-1}$  is increasing,  $\Delta b^1 > 0$  and that  $g^1 (x)$  decreases for all  $x > 0$  and thus  $2G^2 (\Delta b^2) - 1 \geq 0$  ( $g^1$  is symmetric around its unique extremum at 0). As regard the effect of the second stage advantage on the first stage efforts, we differentiate both sides of equation (9) with respect to  $\Delta b^2$ . It comes :

$$\begin{aligned} \partial \tilde{e}^1 / \partial \Delta b^2 &= 2g^1 (\Delta b^1) g^2 (\Delta b^2) \delta_a^2 (\Delta U^3 + \varphi) \\ &\quad \times \Theta_1^{-1'} (g^1 (\Delta b^1) \delta_a [\Delta U^2 + \varphi + \delta_a (2G^2 (\Delta b^2) - 1) (\Delta U^3 + \varphi)]) \quad (> 0). \end{aligned}$$

The second period bias has a disincentive effect on the second period efforts while it increases first period efforts.

### Proof of Theorem 2

*The proof has two parts.*

---

<sup>21</sup>One may observe that the symmetry of the density function around zero allows to maintain the symmetry of the equilibrium, using the following property :  $g^p (\Delta b^p) = g^p (-\Delta b^p)$ ,  $\forall p = 1, 2$ .

a) *It is a MPE.* Let us show that university  $j$  has no incentive to deviate. If  $j$  proposes a higher wage at any stage, two cases may arise. This increase is not sufficient to attract the ranked-first agent. Then  $j$  gets higher costs but returns remain unchanged. If the increase is sufficiently high, university  $j$  attracts the ranked-first agent but, according to *ii*),  $j$  expected payoffs are lower. If  $j$  decreases any proposed wage, then the ranked-second agent has an incentive to deviate outside the university system. The does not fulfill the position and gets a lower return according to A3.1. Thus,  $j$  has no incentives to move. Let us show now that university  $i$  has also no incentive to deviate. If  $i$  increases any wage, revenues remain the same while costs increase. If  $i$  decreases wages of any increment, then  $j$  reacts by setting a wage which allows it to hire the ranked-first agent and increase its returns. This would of course sharply decrease  $i$ 's payoffs. Then  $i$  has no incentive to move either.

b) *No other MPE exists.* Excluding the above mentioned MPE equilibrium, there are four possible situations for any given stage  $p$  (which can be treated independently) which can be categorized by comparing the wages proposed by  $i$  and  $j$  to the one proposed at  $\hat{\sigma}^t$ . 1/ Both universities propose higher wages at stage  $p$  : if  $j$ 's proposition is not sufficient to attract the raked-first agent, then it has a incentive to reduce its proposition. If it is sufficient to attract the ranked-first agent, then  $j$  has also an incentive to reduce its proposition. That is because, then its payoffs are lower than just setting its proposition to  $\underline{g}^{p,t}$  and hire the ranked-second. If  $i$  was setting its wage proposal at  $\bar{s}^{p,t}$ , this would be verified (condition *ii*). Now that  $i$  makes an even higher proposal, this is clearly also verified. Then  $j$  has an incentive to deviate. 2/ University  $j$  proposes a higher wage at stage  $p$  and  $i$  a lower wage : if  $j$ 's proposition is not sufficient to attract the best scientist. Then  $j$  has clearly an incentive to deviate. If it is sufficient, then  $i$  has an incentive to increase its proposition so that it will read the threshold given in *ii*), that is when  $j$  will prefer reducing its proposition and aim at hiring the ranked-second agent. 3/ Both universities propose lower wages at  $p$  : then  $j$  for sure can not hire any agent and gets lower payoffs according to A3.2. University  $j$  deviates. 4/ University  $j$  proposes a lower wage and  $i$  a higher at  $p$  : then  $j$  for sure can not hire any agent and gets lower payoffs according to A3.2 and  $i$  pays a higher salary for no compensation. University  $j$  deviates. In all situations which differ from the MPE, at least one university deviates. Then, there is no other equilibrium.  $\square$

### Computation of the optimal wages

The program of the social planner can thus be rewritten as follows :

$$\max_{\hat{\mathbf{S}}^t, \forall t > 0} \Phi = \sum_{\tau=0}^{\infty} \delta^\tau (2\phi [f^1(\tilde{e}_1^\tau) + f^2(\tilde{e}_2^\tau) + f^3(\tilde{e}_3^\tau) + 3\varepsilon] + 3\phi\Delta b^{p,\tau} - [2s^{1,\tau} + \hat{s}^{2,\tau} + \bar{s}^{2,\tau} + \hat{s}^{3,\tau} + \bar{s}^{3,\tau}]).$$

Given the specifications introduced in Section 4, we have  $\Theta_1(e) = \frac{c}{a}e$  and  $\Theta_2(e) = \frac{c}{\mu a}e$ . Moreover, one obtains :  $f^1 \circ \Theta_1^{-1}(x) = \frac{a^2}{c}x$  and  $f^2 \circ \Theta_2^{-1}(x) = \frac{(\mu a)^2}{c}x$ . Then  $\Phi$  becomes :

$$\Phi = \sum_{\tau=0}^{\infty} \delta^\tau \left( \begin{array}{l} 2\phi \left[ \frac{a^2}{c} \delta_a g(\Delta b^{p,\tau}) (\ln \hat{s}^{2,\tau} - \ln \bar{s}^{2,\tau} + \varphi + \delta_a [2G(\Delta b^{p,\tau}) - 1] (\ln \hat{s}^{3,\tau} - \ln \bar{s}^{3,\tau} + \varphi)) \right. \\ \left. + \frac{a^2 \mu^2}{c} \delta_a g(\Delta b^{p,\tau}) (\ln \hat{s}^{3,\tau} - \ln \bar{s}^{3,\tau} + \varphi) + 3\varepsilon \right] + 3\phi\Delta b^{p,\tau} - [2s^{1,\tau} + \hat{s}^{2,\tau} + \bar{s}^{2,\tau} + \hat{s}^{3,\tau} + \bar{s}^{3,\tau}] \end{array} \right)$$

In the long run, when  $\tau \rightarrow \infty$ , as shown in Corollary 5, the cumulative advantage becomes stationary  $\Delta b^{p,\tau} = b$ . Then, the optimal wages also become stationary  $\hat{s}^{p,\tau} = \hat{s}^p$  and  $\bar{s}^{p,\tau} = \bar{s}^p, p = 2, 3$ . The first order conditions of the social planner's program computed with respect to the lowest wages, lead to negative values. Then the social planner saturates the participation constraint of the ranked-second agents at the two stages considered :  $\underline{s}^2 = \exp(\widehat{W}^2)$ , and  $\underline{s}^3 = \exp(\widehat{W}^3)$ . After some combinations and simplifications, the first order conditions computed with respect to the highest wages lead to the equilibrium wages given in (19) and (20).

## References

- Allison P.D., Stewart J.A., 1974, "Productivity differences among scientists : Evidence for accumulative advantage", *American Sociological Review* 39 (4), 596-606.
- Arora A., David P.A., Gambardella A., 1998, "Reputation and competence in publicly funded science : Estimating the effects on research group productivity", *Annales d'Economie et de Statistique* 49/50, 163-198.
- Barabási A.L., Jeong H., Néda Z., Ravasz E., Schubert A., Visek T., 2001, "Evolution of the social network of scientific collaborations", working paper, cond-mat/0104162.
- Brock, W. A., Durlauf, S. N., 1999, "A formal model of theory choice in science", *Economic Theory* 14, 113-130.
- Carmichael, H.L., 1988, "Incentives in Academics : Why is there tenure ?", *Journal of Political Economy* 96 (3), 453-472.
- Carayol N., 2003, "Objectives, agreements and matching in science industry collaborations : Reassembling the pieces of the puzzle", *Research Policy* 32, 887-908.
- Carayol N., 2003, "Optimal academic competition in the presence of dynamic incentives and cumulative advantages", mimeo BETA, Université Louis Pasteur.
- Carayol N., 2001, *Properties and failures of Open Science. Essays in the economics of 'imperfect science'*, PhD. Thesis, University of Toulouse 1.
- Carayol N., Dalle J-M., 2004, "Sequential problem choice and the reward system in the Open Science", mimeo BETA Université Louis Pasteur.
- Chiappori P-A., Salanié B., Valentin J., 1999, "Early starters versus late beginners", *Journal of Political Economy* 107 (4), 731-760.
- Chung K.H., Cox R.A., 1990, "Patterns of productivity in the finance literature : A study of the bibliometrics distribution", *The Journal of Finance* 45 (1), 301-309.
- Cole S., 1970, "Professional standing and the reception of scientific discoveries", *American Journal of Sociology* 76, 286-306.
- Cole J.R., Cole S., 1973, *Social Stratification in Science*, University of Chicago Press, Chicago.
- Cox R.A., Chung K.H., 1991, "Patterns of research output and author concentration in the economic literature", *Review of Economics and Statistics* 73 (4), 740-747.
- David P.A., 1985, "Clio and the economics of QWERTY", *American Economic Review, Papers and Proceedings* 75, 332-337.

- David P.A., 1994, "Positive feedbacks and research productivity in science : Reopening another black box", in Granstrand O. (ed.), *Economics of Technology*, Elsevier, Amsterdam.
- Diamond A.M., 1996, "The Economics of Science", *Knowledge and Policy* 9 (2/3), 6-49.
- Diamond A.M., 1986, "The life-cycle research productivity of mathematicians and scientists", *The Journal of Gerontology* 41 (4), 520-525.
- Diamond A.M., 1984, "An economic model of the life-cycle research productivity of scientists", *Scientometrics* 6 (3), 189-196.
- Garner C.A., 1979, "Academic publication, market signaling, and scientific research decisions", *Economic Inquiry* 17, 575-584.
- Hansen W.L., Weisbrod B.A., Strauss R.P., 1978, "Modeling the earnings and research productivity of academic economists", *Journal of Political Economy* 86 (4), 729-741.
- Lach, S., Schankerman M., 2003, "Incentives and invention in universities", NBER Working Paper #9267.
- Laffont J.J., Tirole J., 1988, "Repeated auctions of incentive contracts, investment, and bidding parity with an application to takeovers", *Rand Journal of Economics* 19, 516-537.
- Lazear E.P., 1996, "Incentives in basic research", *Journal of Labor Economics* 15 (1), 167-197.
- Levin S.G., Stephan P.E., 1991, "Research productivity over the life cycle : Evidence for academic scientists", *American Economic Review* 81 (1), 114-132.
- Maskin, E., Tirole J., 2001, "Markov perfect equilibrium, I : observable actions", *Journal of Economic Theory* 100, 191-219.
- McDowell J.M., 1982, "Obsolescence of knowledge and career publication profiles : Some evidence of differences among fields in costs of interrupted careers", *American Economic Review* 72 (4), 752-768.
- Merton R.K., 1988, "The Matthew effect in science, II : Cumulative advantage and the symbolism of intellectual property", *ISIS*, vol. 79, 606-623.
- Merton R.K., 1968, "The Matthew effect in science", *Science* 159, 56-63.
- Merton, R.C., Merton, R.K., 1989, "Unanticipated consequences of the reward system in science : A model of the sequencing of problem choices", unpublished manuscript.
- Meyer M.A., 1992, "Biased contests and moral hazard : Implications for career profiles", *Annales d'Economie et de Statistique* 25/26, 165-187.
- Meyer M.A., 1991, "Learning from coarse information : Biased contests and career profiles", *Review of Economic Studies* 58, 15-41.
- Milgrom P., Roberts J., 1988, "An economic approach to influence activities in organizations", *American Journal of Sociology* 94, S154-S179.
- Murphy L., 1973, "Lotka's Law in the humanities", *Journal of the American Society for Information Science* 24, 461-462.
- Prendergast C., Topel R.E., 1996, "Favoritism in Organizations", *Journal of Political Economy* 104, 958-78.
- Radhakrishnan T., Kernizan R., 1979, "Lotka's Law and computer science literature", *Journal of the American Society for Information Science* 30, 51-54.

Siow A., 1991, "Are first impressions important in academia?", *The Journal of Human Resources* 26 (2), 236-255.

Stephan P.E., 1996, "The economics of science", *Journal of Economic Literature* 34, 1199-1235.

Stephan, P.E., Levin S.G., 1997, "The critical importance of careers in collaborative scientific research", *Revue d'Economie Industrielle* 79, 45-61.

Weiss Y., Lillard L.A., 1982, "Output variability, academic labor contracts, and waiting times for promotion", in R.G. Ehrenburg (ed.), *Research in Labor Economics* 5, 157-188.

Zivney T.L., Bertin W.J., 1992, "Publish or perish : What the competition is really doing", *The Journal of Finance* 47 (1), 295-329.

Zuckerman H.A., Merton R.K., 1972, "Age, aging, and age structure in science", in M.R. Riley, M. Johnson et A. Foner (eds.), *A sociology of age stratification : Aging and society*, vol. 3, reprinted in R.K. Merton, 1973, *The Sociology of Science*, N.W. Storer (ed.), University Chicago Press, Chicago, 497-559.