Optimal Dynamic Incentives
in the Academic Competition

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Abstract: In this paper, we introduce a model of academic competition which allows us to discuss the provision of dynamic incentives to scholars along the different steps of their career paths. Agents who initially produced more are hired in universities which offer both higher wages and intrinsically more productive research positions (accumulative advantage). We explore the optimal provision of incentives when one of the following two policy instruments are (even partially) available to the regulator: the wages and the accumulative advantage.

Key words: Dynamic incentives, Cumulative advantages, Biased sequential tournaments, Academic competition.

JEL classification: C73, H00, J44, J41, J72.

1 Introduction

This paper concerns the provision of incentives to academic scholars for scientific accomplishments[1,2]. The design of an optimal employment contract is particularly complicated in the academic context since abilities and efforts are not directly observable, production records are usually delayed and scientific quality and significance are difficult to grasp. Therefore, in the academia, most employment contracts are not based on such variables. Our point of departure is that incentives provided to scholars are mainly dynamic (or implicit), that is based on the expected impact of present efforts on the continuation career (as in standard career concern models, e.g. Holmström, 1982). Scholars seek for *credit* (Merton, 1957) or scientific reputation in their peer community. The higher their reputation, the higher their chance to be proposed positions with increased salaries and prestige (Dasgupta and David, 1994).

In its simplest expression across national and institutional specificities, the standard academic career path follows a succession of three steps. First, agents are Ph.D. candidates, then they become assistant professors an last they hold a full professorship before they get retired. In this paper, we propose a simple model of academic competition in which two agents follow such a path. The competition comes in the process as follows. Positions provide different utility levels (due to differences in wages or non monetary satisfactions) and preferred positions are offered to the best ranked agent in the previous stage. There are two implicit assumptions here. Preferred positions are offered to most productive scholars. If one needs systematic evidence to support such assumption, one can refer to the study of Ault, Rutman and Stevenson (1978, 1982) on the mobility patterns of more than 3,800 economists. They showed that further "upward mobility" (mobility associated with an increase in the quality of the institution) is mainly explained by past publications. Secondly, only ordinal information (ranking) on the previous

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[1]Several modelling attempts have considered other dimensions of the academic institution. The model introduced in Carmichael (1988) intends to explain the tenure system. Merton and Merton (1989) describe the optimal timing scheme for solving a set of scientific problems. Lazear (1997) models funding agencies. Brock and Durlauf (1999) propose a model of discrete choice of scientific theories when agents have an incentive to conform to the opinion of the community. Carayol and Dalle (2004) propose a model of scientific knowledge accumulation over an increasing set of scientific areas. Lach and Schankerman (2003) model the licensing of scientific discoveries.

[2]We are not concerned in this paper with the teaching part of the job. The issue of the allocation of efforts between the two tasks may be analysed in a multitasking static framework (Gautier and Wauthy, 2004) or in a dynamic one in the spirit of Dewatripont, Jewit and Tirole (1999).

stage is available and/or relevant. This might be justified from an informational basis, it can also be justified by observing that universities do indeed organize tournaments to fulfill given available positions (though the precise organization of the tournament frequently differs across universities and national systems). If one admits these two assumptions, therefore everything works as if a sequence of tournaments is designed at the whole academic system level.

In this context, it is worthily noticing that universities, as well as the research positions they propose, are quite heterogeneous in terms of their associated productivity. Prestigious institutions are endowed with heavy instrumentation equipment that less established ones cannot afford (Stephan, 1996). More: the reputation of the hosting institution may signal researchers abilities, enabling them both to raise more funds and to diffuse their results in the scientific community more widely and quickly (Cole, 1970). The detailed study of Cole and Cole (1973) on a sample of physicists, highlighted the importance of the quality of the university department for productivity. Long (1978) showed that, for scientists moving into their first academic position, publication levels are not immediately affected by the prestige of the new department. However, after the third year appointment, scientists productivity tends to be directly related to the prestige level of the new department and unrelated to the old one. Clearly it is not an easy task to get rid of the selection bias that would make best institutions attract best skilled researchers, but there is plenty of anecdotal evidences from day life in universities that tend to show a competition bias does exist in the academe.

If the most productive positions are also the preferred ones (which is quite likely to arise) in the sequential tournament context, then initial successes give access to a better productivity and in turn favour later successes. In other words, the competition over the whole career path tends to be dynamically biased in the sense of Meyer (1992), i.e. biased by a specific form of accumulative advantage (Merton, 1988; David, 1994). Empirical evidence tends to support that an accumulative advantage exists in science: Allison and Stewart (1974) and Allison et al. (1982) found an increasing (linear) dispersion through time of research productivity (for both the number of research publications in the preceding five years and the number of citations to previously published work) between scientists belonging to the same cohort in several fields (Physics, Chemistry and Mathematics).

It is not the purpose of this paper to analyze the formation of the differentials in wages proposed by academic institutions nor it is to find the (long run) factors that make some universities provide to their scholars a much more favorable research environment[3]. We take these for granted and rather focus on their impact on the incentives provided to scholars at the different stages of the career. We specifically study the incentive properties of the cumulative and initial advantages so as to provide a theoretical framework to empirical studies which argued biased competition distorts the academic incentive structure (e.g. Zuckerman and Merton, 1972). We find that the initial advantage (static bias) deters efforts (as in the standard tournament literature) while the accumulative advantage (dynamic bias) improves the early career efforts because it generates dynamic incentives.

Academic tournaments are neither fully designable competitions as the ones set by shareholders or managers in organizations, nor fully exogenous as for reputation based external labour market competition. Rather, the academic system seems lies somewhere in between: the regulator (the state or public agencies) has some degree of control which varies across national systems. For instance the regulator may partially intervene on wages through some regulatory decisions. The regulator may also modify the gradient of universities advantages by playing on the allocation of public funds among universities or labs (which regulators actually usually do). We challenge that issue in the present paper by studying the optimal wage differentials and optimal accumulative advantage.

The paper is organized as follows. The model is presented in the next section. The third section is dedicated to agents' behaviors under such biased competition. The fourth section computes the optimal compensation schemes and the optimal cumulative advantage. The last section concludes by summing up the main results and discussing some of their implications.

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[3]Is the specific purpose of a another paper (Carayol, 2005) which focuses on the competition between universities in the recruitment of agents and the endogenous emergence of an accumulative advantages in the long run.

2 The model

2.1 Basic settings

Let us define the academic researchers population as a generation of two agents whose careers last three periods. Let $p \in \{1, 2, 3\}$ denote the career steps at which the agents are either Ph.D. candidates, junior (assistant professor or researcher), or senior (full professor or researcher). At instant 0, a fixed cohort C arrives, composed of two researchers i and j. The academic research system is composed of two research institutions, say universities (or departments or laboratories) h and l. Research production is supposed to be additively separable in efforts over the first two periods of activity[4]. At stage p, it is given by the following expression:

$$y_k^p = f^p\left(e_k^p\right) + \beta_k^p + \varepsilon_k^p \tag{1}$$

where scientific knowledge production is a function of efforts spent at stage p of their career e_k^p, with $f^p\left(\cdot\right)$ a positive and increasing function the derivative of which gives the productivity of efforts at the first two steps of the career (Ph.D. and junior researcher: $p \in \{1, 2\}$). $f^p\left(\cdot\right)$ is assumed to be weakly concave. ε_k^p is the random specific shock that affects agent k production at stage p. Let us assume these shocks are *iid* across agents and periods and distributed around zero. We define $\Delta\varepsilon^p$, as the difference between the individual random shocks at stage p: $\Delta\varepsilon^p \equiv \varepsilon_i^p - \varepsilon_j^p$. The distribution of these random variables is assumed to identical across periods. The cumulative distribution function is denoted $G\left(\cdot\right)$ and its density function is $g\left(\cdot\right)$. The latter is assumed to be unimodal, continuously differentiable and strictly positive over $[-\infty, \infty]$ and symmetric around its unique maximum attained at 0. β_k^p is the production premium due to the agents' context of work: It is an attribute of the university$\times$ career stage. By definition we have: $\beta_k^p \in \left\{\beta_h^p, \beta_l^p\right\}, k = i, j; p = 1, 2$, with $\beta_j^p \neq \beta_i^p$. This simply means that agent k's production will be improved at step p by some β_u^p just because k is working at research institution $u \in \{h, l\}$. Without loss of generality, we assume that: $\beta_h^p \geq \beta_l^p, \forall p = 1, 2$. Let us define $\Delta\beta^p \equiv \beta_h^p - \beta_l^p$, the differences in production at the two first steps uniquely due to the immediate environment of work.

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[4]There is no incentive in our model in the last period. We do not assume that senior faculty do not exert any effort, but that their behaviours are driven more by intrinsic motives than by external incentives. Therefore, last period academic accomplishments are exogenous in our model ($y_k^3 = y^3$).

The instantaneous net utility of agent $k \in C$ is given by the function $W\left(s_k^p, e_k^p\right)$ which is assumed to be additively separable between the disutility of efforts and the utility of revenues:

$$W_k^p = W\left(s_k^p, e_k^p\right) = U(s_k^p) - V(e_k^p) \tag{2}$$

with $U\left(\cdot\right)$ the instantaneous utility of wages such that agents are assumed to be intra-period risk averse: It is continuous, concave and twice differentiable. The disutility of efforts $V\left(\cdot\right)$ is continuous convex and twice differentiable. More precisely, we make the standard following assumptions: $U' > 0, U'' \leq 0$, $\lim_{s \to 0} U(s) = -\infty$, and $V(0) = V'(0) = 0$; $V' > 0$ and $V'' \geq 0$ over $\Re^{+*}$ and $\lim_{e \to \infty} V'(e) = \infty$. The whole career net utility function is assumed to be additively separable between the three periods of the career. We also assume that agents do not have access to the financial market so that they can neither save nor borrow and thus shall consume their whole revenue received at each period considered. Thus, the actualized total net utility is given by:

$$\Pi_k = \Pi\left(\mathbf{e}_k, \mathbf{S}\right) = \sum_{p=1\ldots3} \delta_a^{p-1} W_k^p \tag{3}$$

with δ_a the agents' actualization factor and $\mathbf{e}_k$ the vector describing agent $k \in C$ efforts over his whole career, that is namely: $\mathbf{e}_k = \left(e_k^1, e_k^2, e_k^3\right)$. The wages offered by the research system to cohort C is described by the vector $\mathbf{S} \equiv \left(s^1, \overline{s}^2,\ \underline{s}^2, \overline{s}^3, \underline{s}^3\right)$, with $\overline{s}^p$ ($\underline{s}^p$) denoting the wage offered at stage p of the career to the preceding stage winner (loser). The first step wages are assumed to be unique s^1.

2.2 Competition biases and timing

The main idea in the sequencing of academic steps is that the first winner can choose the junior position that gives an advantage to obtain again the best senior position. This is the accumulative advantage. There is also an initial static advantage given to one of the Ph.D. candidates. Let us state these formally as rules of attribution of the production premium differentials $\Delta\beta^p$ in the following assumption.

Assumption 1. *The attribution of the production difference $\Delta\beta^1$ at the first stage is independent of the competition. As such, the first period bias is a static bias of the academic competition. The ranked first agent at the end of the first stage benefits from the second stage*

bias $\Delta\beta^2$. *The second stage bias is thus a dynamic bias. Formally, i benefits from it according to* $\beta_i^2 = \beta_j^2 + \Delta\beta^2 \times \text{sign}\left[y_i^1 - y_j^1\right]$.

Let us now specify the exact timing of the game:

- at 0: Cohort C arrives. The universities publicly announce their respective payment schemes, that is the initial payment, the wages they will offer for junior researchers positions and the one they will propose for senior researchers positions (summed up in vector $\mathbf{S}$). The productivity levels associated with each position is made public among scientists belonging to the arriving cohort. The members of C are arbitrarily allocated to the Ph.D. position in the two universities. Conditional to the information on the initial allocation, they can choose to exit;

- at 0.5: The Ph.D.s choose their effort levels;

- at 1: The first round of the competition ends up. Ranking is public knowledge. The most productive researcher chooses the junior position (i.e. the university), while the other researcher either accepts the remaining junior researcher position or chooses the outside option;

- at 1.5: The agents choose the effort level of their second career cycle period;

- at 2: The second round competition ends and the ranking is public knowledge. The winner chooses the senior position he or she ranks first. The loser decides whether he accepts the other senior position or exits;

- at 2.5: Agents are in the last period before their careers end. In the setting of the present paper, the production levels are exogenous and they receive the wages associated with their positions;

- at 3: they retire.

3 Researchers' behaviors and the incentive properties of biased competition

We now concentrate on the derivation of scientists equilibrium behaviors in the presence of biases. Since agents act in a standard backward induction manner, we first study the second stage competition and next focus on the first stage competition. Proposition 1 summarizes comparative statics results concerning how wages differentials and biases affect equilibrium efforts.

3.1 Second stage Nash equilibrium

At the second stage, agent i chooses her effort level e_i^2, given j's (e_j^2) in order to maximize her expected net utility from then. Thus, i's program at the beginning of stage 2 consists in maximizing the expected net utility actualized at the second career stage, that is[5]

$$P\left(y_i^2 > y_j^2\right) \times \delta_a U\left(\bar{s}^3\right) + \left[1 - P\left(y_i^2 > y_j^2\right)\right] \times \delta_a U\left(\underline{s}^3\right) - V\left(e_i^2\right). \tag{4}$$

It is equal to the probability of winning the second tournament times the utility she receives then $(U\left(\bar{s}^3\right))$, plus the probability to loose the tournament times the utility received if he looses it $(U\left(\underline{s}^3\right))$, net of the disutility of efforts. An identical program could be written for j. The simultaneous resolution of the two programs detailed in the Appendix leads to a unique Nash equilibrium which is symmetric and given by

$$\tilde{e}^2 = \Theta_2^{-1}\left(\delta_a\left(\Delta U^3\right) g\left(\Delta\beta^2\right)\right), \tag{5}$$

with function $\Theta_2\left(\cdot\right)$ defined by $\Theta_2\left(x\right) = V'\left(x\right)/f^{2\prime}\left(x\right)$ and with $\Delta U^3 = U\left(\bar{s}^3\right) - U\left(\underline{s}^3\right)$ the differences in third stage utility. This function is null at 0 and strictly increasing from then (and thus so is its inverse function $\Theta_2^{-1}\left(\cdot\right)$).

3.2 First stage subgame perfect Nash equilibrium

We now turn to the first period agents' behaviors. Agent i objective is to determine her first period effort level in order to maximize expected net utility, that is

$$\tilde{e}_i^1 = \arg\max\left\{\Pi_i\right\}. \tag{6}$$

This maximization program is different from the second period one since the first period success influences the second stage competition. The unique and symmetric subgame perfect Nash equilibrium (again detailed computations are in the Appendix) is given by:

$$\tilde{e}^1 = \Theta_1^{-1}\left(g\left(\Delta\beta^1\right)\delta_a\left[\Delta U^2 + \delta_a\left(2G\left(\Delta\beta^2\right) - 1\right)\left(\Delta U^3\right)\right]\right) \tag{7}$$

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[5]We ommit second stage utility which is independant of second stage efforts.

with $\Theta_1(\cdot)$ such that $\Theta_1(x) = V'(x)/f^{1'}(x)$ and Θ_1^{-1} is increasing.

Comparative statics results on both second and first stage equilibrium efforts are summed up in Proposition 1 below. It confirms the results of several empirical according to which the accumulative advantage stimulates early career efforts while it diminishes late career efforts.

Proposition 1. *Agents' equilibrium efforts are unique and symmetric at each stage of the career cycle. Equilibrium efforts increase with the differences in utility provided by positions at the remaining stages of the career (ΔU^2 and ΔU^3). The accumulative advantage ($\Delta\beta^2$) increases first period equilibrium efforts while it decreases the second period efforts. The first stage advantage ($\Delta\beta^1$) decreases first period efforts.*

Proof. See the Appendix.

4 Optimal academic competition

We assume that the social surplus created by the academic activity is simply obtained, through function $\Phi(\cdot)$, by the actualized sum of the individual productions times a given parameter ψ. This parameter ($\psi > 0$) gives the unitary social value of scientific knowledge which is assumed to be homogenous (or normalized). Thus the total surplus actualized at the first period generated by a given cohort is given by:

$$\Phi(\mathbf{Y}) \equiv \psi \sum_{p=1,2,3} \delta^{p-1} \left(y_i^p + y_j^p \right), \tag{8}$$

with δ the social actualization factor and $\mathbf{Y}$ the vector of productions of cohort C. The expected net surplus per capita of a given cohort is given by the following expression:

$$\pi(\mathbf{Y}, \mathbf{S}) \equiv \frac{1}{2} \mathrm{E}\left[\Phi(\mathbf{Y})\right] - \frac{1}{2} C(\mathbf{S}, \delta), \tag{9}$$

with $C(\mathbf{S}, \delta)$ the total costs associated with the productions $\mathbf{Y}$ which are actualized at the first period by means of the social discount factor δ.

From now we specify the functions $V(\cdot)$ and $f^p(\cdot)$ in accordance with their generic properties given in Section 2. The disutility function is assumed to be quadratic in efforts: $V(e) = \frac{1}{2}ce^2$.

The production functions of scientific knowledge are assumed to be linear in efforts, with: $f^2(e) = \mu f^1(e) = \mu ae$. The parameter μ gives the increase rate in agents' productivity between the first two periods of their career (Ph.D./junior): $\mu = f^2(e)/f^1(e)$. If we have $\mu > 1$, the agents' efforts productivity increases through the career path. Under these assumptions, one gets linear $\Theta_1(\cdot)$ and $\Theta_2(\cdot)$ with: $\Theta_1(e) = \frac{c}{a}e$ and $\Theta_2(e) = \frac{c}{\mu a}e$. Then their inverse functions are also linear in efforts. Let also the $\Delta \varepsilon^d$ be identically distributed across the different periods of the career, that is $g^d(\cdot) = g(\cdot), \forall d$. Let us also simply define the costs function as $C(\mathbf{S}, \delta) = 2s^1 + \delta\left(\overline{s}^2 + \underline{s}^2\right) + \delta^2\left(\overline{s}^3 + \underline{s}^3\right)$.

The regime of outside options of scholars considers both that agents may go out at each period, using then the (increasing) information collected so far. So as to simplify the notations, we assume agents compare the second stage outside option with the current period utility (and not with the whole career expected utility flow)[6]. Formally, one obtains three arbitrage conditions for remaining in the academia at each period.

$$\mathrm{E}\left[W_k^p | \Delta\beta^p\right] \geq \overline{W}^p, p = 1, 2, 3 \tag{10}$$

with $\overline{W}^p$ the outside option utility at period p. Since the expected utility of the agents that do not benefit from the current period bias at each period is lower, we only need to concentrate on this agent situation at the different stages.

4.1 The optimal compensation scheme

This section is dedicated to the study of the optimal compensation scheme. Wages may be tuned by a central planner in countries where the national research system is controlled by the State (e.g. France). In the model, the programme of the regulator is simply the maximization of the expected social surplus subject to the incentive constraint and the three participation constraints. It is given by

$$\max_{\mathbf{S}} \ \pi(\mathbf{Y}, \mathbf{S}), \tag{11}$$

subject to the incentives constraints given in (5) and (7), and subject to the participation

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[6] The (standard) implicit assumption here is that past scientific production has no effect on the outside option. More: once out of the academe, there is no way back.

constraints given in (10).

Detailed computations can be found in the Appendix which derives the optimal wages. In short, the optimal wages proposed to ranked second agents will saturate the participation constraints since they both increase costs and have a negative impact on incentives provided to scholars at the preceding stages of activity (by lowering the differences between the satisfaction between winning and loosing the tournaments). Therefore, we have: $(\underline{s}^p)^* = \exp\{\bar{W}^p\}, p = 2, 3$. The wages proposed the ranked first agents are set so as to optimally balance incentive effects of wages and its costs. The ranked first wages at the second and third period periods are:

$$\left(\bar{s}^2\right)^* = 2\frac{\psi\mu a^2\delta_a}{c\delta}g\left(\Delta\beta^1\right), \tag{12}$$

$$\left(\bar{s}^3\right)^* = \frac{\psi\mu a^2\delta_a}{c\delta}\left(\frac{\delta_a}{\delta}g\left(\Delta\beta^1\right)\left(2G\left(\Delta\beta^2\right) - 1\right) + g\left(\Delta\beta^2\right)\right). \tag{13}$$

Finally the first period unique compensation s^1 saturates the first participation constraint: $s^1 = \exp\{\bar{W}^1\}$.

Comparative statics performed on the wages proposed to ranked first agents lead to the Proposition 2 below.

Proposition 2. *The optimal second and third periods wages proposed to ranked first agents increase with the social value of knowledge (ψ), the productivity of efforts (a), the increase of productivity across the career (μ) and the agents actualization factor (δ_a). They decrease with the first period (static) bias $\left(\Delta\beta^1\right)$, the disutility of efforts (c) and the social discount factor (δ). The third period highest wage also decreases with the second period (dynamic) bias $\left(\Delta\beta^2\right)$ when the first bias is sufficiently high: that is when $g\left(\Delta\beta^1\right) < \frac{-\delta}{2\delta_a}g'\left(\Delta\beta^2\right)/g\left(\Delta\beta^2\right)$.*

Proof. See the Appendix.

4.2 The optimal accumulative advantage

This subsection concentrates on the use of an alternate policy instrument. The regulator can not play on wages but has some possibilities to tune the dynamic bias[7]. In short the

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[7]As regard the first period static bias $\Delta\beta^1$, it is clear from the preceeding section that it decreases incentives and should optimaly be null.

issues addressed here are the following. Is there to be found some optimal quality gradient between research institutions which maximizes researchers' incentives? If so, how does such accumulative advantage should be tuned according to more specific settings?

In order to compute that optimal level of accumulative advantage, the equilibrium efforts levels may be expressed in terms of the dynamic bias. Thus, the expected net social surplus per capita may also be written as a function of the dynamic bias $\tilde{\pi}\left(\Delta\beta^2\right)$ while wages are assumed to be fixed exogenously. The optimal dynamic bias is the one that maximizes the expected social surplus. It is given by the following programme:

$$\max_{\Delta\beta^2} \left\{\tilde{\pi}\left(\Delta\beta^2\right)\right\}, \tag{14}$$

subject to the incentives constraints given in (5) and (7), and subject to the participation constraints given in (10).

In order to study this optimal bias more precisely, $\Delta\varepsilon^p$ is assumed to be normally distributed and centered: $\Delta\varepsilon^d \sim N\left(0,\sigma\right)$[8]. Using such specified expressions of g and g', we obtain the following expression of the optimal bias (see computation details in the Appendix):

$$\left(\Delta\beta^2\right)^* = 2\delta_a g\left(\Delta\beta^1\right)\sigma^2/\delta\mu^2. \tag{15}$$

Proposition 3 follows.

Proposition 3. *There is a unique optimal dynamic bias $\left(\Delta\beta^2\right)^*$ which maximizes the social surplus. Under standard specifications, increases with the variance of differences in production shocks and the agents' discount factor. It decreases with the social discount factor, with the growth rate of agents' productivity through their careers and with the initial inequalities.*

Proof. See the Appendix.

Thus, the optimal dynamic bias increases with the agents discount factor (*i.e.* decreasing with their preference for present) and decreases with the social discount factor. Because the dynamic bias tends to stimulate the first period efforts, it is quite intuitive that the optimal

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[8] Which would for instance naturally arise if the shocks ε^d are themselves is assumed to be normally distributed and centered: $\varepsilon_i^p \sim N\left(0,\sigma_i\right)$, $\varepsilon_j^p \sim N\left(0,\sigma_j\right)$; with $\sigma_i + \sigma_j = \sigma$.

bias increases with the social preference for the present. It also increases with the variance of random shocks differences affecting production. This indicates that the more uncertain the research activity is, the higher the socially optimal bias. Moreover, the optimal bias decreases with the growth rate of productivity through the career path. That may be explained as follows. Knowing that the dynamic bias tends to increase the first period efforts and to diminish the second period ones, the higher (lower) the youth's productivity, the higher (lower) the optimal bias. Lastly, the higher the inequalities due to the initial conditions, the less should be the dynamic bias.

5 Conclusion and discussion

In this paper we have introduced a dynamic model of academic competition aiming to capture the dynamic incentives offered to scholars along the career path in the presence of an accumulative advantage affecting academic competition. We have suggested that one source of this well-known effect lies in the employment relationship because research positions are intrinsically unequally productive and because the allocation of the best ones is based on past scientific production. This way, the academic competition is dynamically biased. We extensively studied the optimal setting of this competition, presenting the optimal initial career advantage, the optimal wages and the optimal mid-career accumulative advantage. The results obtained are summed up in Proposition 1 to Proposition 3. Let us now discuss some of their potential implications.

We showed that it is strictly sub-optimal that a static bias affects competition in the early career. This standard result in the sequentiel tournament litterature simply arises because it lowers efforts of agents whether they are advantaged or disadvantaged by it. In the academic context, biases of such kind may intervene because of the quality differences in the initial positions (at Ph.D. level). Therefore, developing for instance shared doctoral training opened to Ph.D. candidates in secondary research institutions would globally increase incentives (regardless the training benefits).

In research systems where the central planner can set up wages (e.g. in France), incentives can be enhanced by using that variable. We have computed the optimal compensation scheme

under standard specifications of the production function, of the shocks distribution, of the utility and of the disutility functions. What is important for the central planner is then to set winners' wages in order to optimally balance the previous period(s) incentive effects and the costs of wages. The wages offered to the ranked second agents should only guarantee that scientists refuse outside options.

If wages are not modifiable by the central planner, incentives can be improved by tuning the gradient of accumulative advantages provided to agents in accordance with some factors which may for instance vary with disciplines. It should typically decrease with the growth rate of agents' productivity through their career (which can be interpreted as a learning by doing since it is not intentional here). Moreover, the more risky the research activity, the higher the socially optimal cumulative advantage. Lastly, the cumulative advantages affecting scientific productivity should be reduced when initial treatment is unequal (for example due to the quality of the doctoral training).

Appendix

Computation of the second stage Nash equilibrium

The first order condition of program (4) is

$$\delta_a \left(\Delta U^3 \right) \times \partial P \left(y_i^2 > y_j^2 \right) / \partial e_i^2 = \partial V \left(e_i^2 \right) / \partial e_i^2.$$

Notice that the probability that i wins the second tournament is given by

$$P \left(y_i^2 > y_j^2 \right) = P \left(f^2 \left(e_i^2 \right) + \Delta \beta^2 + \Delta \varepsilon^2 > f^2 \left(e_j^2 \right) \right) = \left[1 - G \left(f^2 \left(e_j^2 \right) - f^2 \left(e_i^2 \right) - \Delta \beta^2 \right) \right].$$

When differentiating that expression with respect to i's efforts, one obtains:

$$\partial P \left(y_i^2 > y_j^2 \right) / \partial e_i^2 = f^{2\prime} \left(e_i^2 \right) \times g \left(f^2 \left(e_j^2 \right) - f^2 \left(e_i^2 \right) - \Delta \beta^2 \right).$$

Introducing this expressions in the first order condition, one gets:

$$f^{2\prime} \left(e_i^2 \right) \times g \left(f^2 \left(e_j^2 \right) - f^2 \left(e_i^2 \right) - \Delta \beta^2 \right) \delta_a \left(\Delta U^3 \right) = V' \left(e_i^2 \right).$$

Let us define the function $\Theta_2(\cdot)$, such as $\Theta_2(x) = V'(x)/f^{2\prime}(x)$. This function is defined over $\Re^+$, $\Theta^2(\cdot) : (0,\infty) \to (0,\infty)$. Since $V'(0) = 0$, this function is null at 0 ($\Theta(0) = 0$). Moreover, having $V' > 0$ and $V'' > 0$ over $\Re^{+*}$, and $f^{2\prime} > 0, f^{2\prime\prime} \leq 0$, it can easily be shown that this function is strictly increasing: $\Theta'_2 > 0$. Thus, its inverse function $\Theta_2^{-1}(\cdot) : (0,\infty) \to (0,\infty)$ is also increasing. Then, one can rewrite the two first order conditions using these new notations:

$$\Theta_2\left(e_i^2\right) = \delta_a\left(\Delta U^3\right) g\left(f^2\left(e_j^2\right) - f^2\left(e_i^2\right) - \Delta\beta^2\right),$$

$$\Theta_2\left(e_j^2\right) = \delta_a\left(\Delta U^3\right) g\left(f^2\left(e_i^2\right) - f^2\left(e_j^2\right) - \Delta\beta^2\right).$$

Given the assumptions formulated so far, these two equations are of the form $e_i^2 = h\left(e_j^2\right)$ and $e_j^2 = h\left(e_i^2\right)$ with $h(\cdot)$ a continuous function over $\Re^+$. Therefore, if an equilibrium exists, it is necessarily symmetric of the form $e_i^2 = e_j^2 = e^2$ such that $e^2 = h\left(h\left(e^2\right)\right)$. This equilibrium then satisfies the following expression:

$$\Theta_2\left(e^2\right) = \delta_a\left(\Delta U^3\right) g\left(\Delta\beta^2\right).$$

Knowing that that $\Theta_2(\cdot)$ is strictly positive, is null at zero, is strictly increasing and that $\lim_{e\to\infty}\Theta_2(e) = \infty$ (since by assumption $\lim_{e\to\infty}V'(e) = \infty$ and $f^{2\prime\prime} \leq 0$), this equation admits a unique solution. Moreover, since $g > 0$, $\Delta U^3 > 0$ and $f^{2\prime} > 0$, this solution is strictly positive. The unique symmetric second stage Nash equilibrium is thus given by

$$\tilde{e}^2 = \Theta_2^{-1}\left(\delta_a\left(\Delta U^3\right) g\left(\Delta\beta^2\right)\right) \qquad \Box.$$

Computation of the first stage subgame perfect Nash equilibrium

Let p_i denote the probability that, if the agent employed by university i has won the first stage competition, he will also win the second stage one. Since the second tournament is influenced by the results of the first one, p_i is a conditional probability. Since the second stage equilibrium efforts are identical (given in 5), that conditional probability is independent of

agents' identity $(p = p_i = p_j)$. It can be computed as follows:

$$p = P(e^2 + \varepsilon_i + \Delta\beta^2 > e^2 + \varepsilon_j) = 1 - P(\Delta\varepsilon < -\Delta\beta^2) = 1 - G\left(-\Delta\beta^2\right).$$

Referring to the assumption that $g\left(\cdot\right)$ is symmetric around 0, one can write: $p = G\left(\Delta\beta^2\right)$.

We note $\Delta\Pi^p$ the surplus of expected utility received from stage p (included) over the remaining career cycle it provides to given that stage p is won and the expected utility given it is lost. Formally it is given by

$$\Delta\Pi^p = \left.\Pi_i^p\right|_{y_i^p > y_j^p} - \left.\Pi_i^p\right|_{y_i^p < y_j^p},$$

with $\left.\Pi_i^p\right|_{y_i^p > y_j^p}$ the expected utility of agent i conditioned on i will win the ongoing p's period tournament.

Using these notations and definitions and after several recombinations, we can rewrite $\Delta\Pi^1$ as follows:

$$\Delta\Pi^1 = \delta_a U\left(\overline{s}^2\right) + \delta_a^2\left[pU\left(\overline{s}^3\right) + (1 - p)U\left(\underline{s}^3\right)\right] - \delta_a U\left(\underline{s}^2\right) - \delta_a^2\left[(1 - p)U\left(\underline{s}^3\right) + pU\left(\underline{s}^3\right)\right].$$

After a few simplifications, it comes:

$$\Delta\Pi^1 = \delta_a\left(\Delta U^2\right) + \delta_a^2\left(2G\left(\Delta\beta^2\right) - 1\right)\left(\Delta U^3\right),$$

with ΔU^2 the difference in utility between having won the first tournament and having lost it. Introducing that expression in the first order condition of the first period maximization program (6), it comes[9]:

$$\Theta_1^{-1}\left(e^1\right) = g\left(\Delta\beta^1\right)\delta_a\left[\Delta U^2 + \delta_a\left(2G\left(\Delta\beta^2\right) - 1\right)\Delta U^3\right],$$

with $\Theta_1\left(\cdot\right)$ defined similarly than with $\Theta_2\left(\cdot\right)$, that is $\Theta_1\left(x\right) = V'\left(x\right)/f^{1\prime}\left(x\right)$. The equilibrium is symmetric and unique for the same reason than for the second period. The final expression

---

[9]One may observe that the symmetry of the density function around zero allows to maintain the symmetry of the equilibrium, using the following property: $g^p\left(\Delta b^p\right) = g^p\left(-\Delta b^p\right)$, $\forall p = 1, 2$.

of the equilibrium efforts (7) follows □.

Proof of Proposition 1: Comparative statics

Since $g(\cdot)$ is assumed to be unimodal and symmetric around zero, $g\left(-\Delta\beta^P\right)$ is maximum for $\Delta\beta^P = 0$ and decreases with $\left|\Delta\beta^P\right|$. Moreover, having $V' > 0, V'' > 0, f^{p\prime} > 0, f^{p\prime\prime} \leq 0$, function Θ'_p is strictly increasing and its inverse function $\Theta_p^{-1}(\cdot) : (-\infty, \infty) \to (0, \infty)$ is also increasing $\forall p = 1, 2$. Thus, at the second stage, the second period Nash equilibrium efforts are maximum when the bias is null and decrease with the bias introduced at this second period $\partial\tilde{e}^2/\partial\left|\Delta\beta^2\right| < 0$. Similarly, the second period equilibrium efforts are increasing with the differences in utility at the last period (ΔU^3).

At the first period, similarly the first period equilibrium efforts decrease with $\left|\Delta\beta^1\right|$ and we obviously have $\partial\tilde{e}^1/\partial\Delta\beta^1 < 0$. This is also due to the fact that $2G\left(\Delta\beta^2\right) - 1$ is positive because the derivative of G, g is symmetric around its unique extremum at 0. For the same reasons, $\tilde{e}^1$ increases with both ΔU^2 and ΔU^3. As regard the effect of the second stage advantage on the first stage efforts, we differentiate both sides of equation (5) with respect to $\Delta\beta^2$. It comes:

$$\partial\tilde{e}^1/\partial\Delta\beta^2 = 2g^1\left(\Delta\beta^1\right)g\left(\Delta\beta^2\right)\delta_a^2\Delta U^3$$
$$\times\Theta_1^{-1\prime}\left(g\left(\Delta\beta^1\right)\delta_a\left[\Delta U^2 + \delta_a\left(2G\left(\Delta\beta^2\right) - 1\right)\Delta U^3\right]\right) \qquad (>0).$$

The second period bias has a disincentive effect on the second period efforts while it increases first period efforts. □

Computation of the optimal wages

When one introduces explicitly the agents calculations in the expected social surplus, it becomes only a function of the compensation scheme. Indeed equation (9) be rewritten as follows:

$$\pi\left(\mathbf{S}\right) = \psi\left(f^1\left(e^1\left(\mathbf{S}\right)\right) + \delta f^2\left(e^2\left(\mathbf{S}\right)\right)\right) + \bar{\gamma} - \frac{1}{2}c\left(\mathbf{S}, \delta\right),$$

with $\bar{\gamma}$ such that $\bar{\gamma} = \bar{\beta}^1 + \delta\bar{\beta}^2 + y^3$, with $\bar{\beta}^p$ $(\forall p = 1, 2)$, the average values of production premia $\bar{\beta}^p = \frac{1}{2}\left(\beta_i^p + \beta_j^p\right)$.

The regulator's programme becomes: $\max_{\mathbf{S}} \pi\left(\mathbf{S}\right)$, subject to the incentives constraints given

in (5) and (7), and subject to the participation constraints given in (10). Given the specifications introduced, scientists' equilibrium efforts are given by:

$$\tilde{e}^1 = \frac{a\mu}{c} g\left(\Delta\beta^1\right) \delta_a \left[\left(\ln \overline{s}^2 - \ln \underline{s}^2\right) + \delta_a \left(2G\left(\Delta\beta^2\right) - 1\right) \left[\ln \overline{s}^3 - \ln \underline{s}^3\right]\right],$$

$$\tilde{e}^2 = \frac{a}{c} g\left(\Delta\beta^2\right) \delta_a \left[\ln \overline{s}^3 - \ln \underline{s}^3\right].$$

When these incentive constraints expressions are introduced in the expected social surplus, it becomes:

$$
\begin{aligned}
\pi\left(\mathbf{S}\right) \;=\; & \psi \frac{a^2\mu}{c} g\left(\Delta\beta^1\right) \delta_a \ln \overline{s}^2 - \frac{1}{2}\delta\overline{s}^2 - \psi\frac{a^2\mu}{c} g\left(\Delta\beta^1\right) \delta_a \ln \underline{s}^2 - \frac{1}{2}\delta\underline{s}^2 \\
& + \left[\psi\frac{a^2\mu}{c} g\left(\Delta\beta^1\right) \delta_a^2 \left(2G\left(\Delta\beta^2\right) - 1\right) + \psi\delta a\mu\frac{a}{c} g\left(\Delta\beta^2\right) \delta_a\right] \ln \overline{s}^3 - \frac{1}{2}\delta^2\overline{s}^3 \\
& - \left[\psi\frac{a^2\mu}{c} g\left(\Delta\beta^1\right) \delta_a^2 \left(2G\left(\Delta\beta^2\right) - 1\right) - \psi\delta\mu\frac{a^2}{c} g\left(\Delta\beta^2\right) \delta_a\right] \ln \underline{s}^3 - \frac{1}{2}\delta^2\underline{s}^3 + \overline{\gamma} - s^1.
\end{aligned}
$$

In that expression, we observe that the wages proposed to ranked first agents both impact positively on social surplus through its incentive effects and negatively due to the costs of wages. Contrarily, the impact of the wages proposed to ranked second agents is unambiguous. It decreases the social surplus both by reducing incentives (by lowering the differences between the satisfaction between winning and loosing the tournaments) and by increasing costs. This simple observation gives us the intuition for setting the optimal wages. The optimum consists here simply in setting the ranked second agents wages so as to saturate their participation constraint, and then in determining the ranked first agents' wages so as to optimally balance incentive effects of wages and its costs. Such computation is eased because the optimal winners' wages will prove to be independent of the losers'. As a matter of fact, the two first order conditions with respect to the winners' wages give us directly the optimal winners' wages in equations (12) and (13). The saturated third period constraint and the second participation constraint, given the saturated third one, gives us the optimal ranked second wages: $\left(\underline{s}^p\right)^* = \exp\left\{\overline{W}^p\right\}, p = 2, 3$. Finally the first period unique compensation s^1 saturates the first participation constraint: $s^1 = \exp\left\{\overline{W}^1\right\}$.

Proof of Proposition 2

The second stage wage proposed to the ranked first agent $\left(\overline{s}^2\right)^*$ and $\left(\overline{s}^3\right)^*$ trivially increase with the productivity parameter a (quadratically), the agents' discount factor δ_a, the unitary value of knowledge ψ and the relative productivity of the juniors compared to the Ph.D. candidates μ and decrease with the social actualization factor δ, the disutility of efforts parameter c and the first bias $\left(\Delta\beta^1\right)$ (since $g'\left(x\right) < 0, \forall x \neq 0$).

The third stage wage proposed to the ranked first agent $\left(\overline{s}^3\right)^*$ exhibits the same parameters effects as the ones of the optimal second period wage of the first winner since $G\left(\Delta\beta^2\right) \geq 1/2$. Moreover, we have $\partial\left(\overline{s}^3\right)^* / \partial\left(\Delta\beta^2\right) < 0$ iff $g\left(\Delta\beta^1\right) < -\frac{\delta}{2\delta_a}\frac{g'\left(\Delta\beta^2\right)}{g(\Delta\beta^2)}$. Since g is decreasing over $\Re^+$, we know that $\left(\overline{s}^3\right)^*$ is also decreasing with $\left(\Delta\beta^2\right)$ we can conclude that the first bias $\left(\Delta\beta^1\right)$ is sufficiently high. $\square$

Computation of the optimal accumulative advantage

In order to perform programme (14), we introduce the expressions of the equilibrium efforts given in (5) and (7) in expression (9). Using the specifications introduced till now, one gets:

$$
\begin{aligned}
\tilde{\pi}\left(\Delta\beta^2\right) = {} & \psi\frac{a^2}{c}\left[\delta_a g\left(\Delta\beta^1\right)\left[\Delta U^2 + \delta_a\Delta U^3\left(2G(\Delta\beta^2) - 1\right)\right]\right. \\
& \left. + \delta\delta_a\mu^2 g\left(\Delta\beta^2\right)\Delta U^3\right] + \bar{\gamma} - \frac{1}{2}C\left(\mathbf{S}, \delta\right).
\end{aligned}
$$

When equalizing to zero the derivative of the right term of this expression, one gets the first order condition of the maximization programme :

$$
-g'\left(\Delta\beta^2\right)/g(\Delta\beta^2) = 2\delta_a g\left(\Delta\beta^1\right)/\delta\mu^2.
$$

Since $\Theta_1\left(\cdot\right)$ and $\Theta_2\left(\cdot\right)$ are linear, the optimal bias is uniquely function of the distribution of shock differences affecting knowledge production $(g(\cdot))$, agents discount factor, the social discount factor, the initial bias and the increase rate on productivity senior/junior. One may observe that the optimal bias is independent of the compensation schemes at the different periods. Using the properties of the Gaussian specifications for g, and recombining the first order condition above gives us directly the expression of the optimal bias:

$$
\left(\Delta\beta^2\right)^* = 2\delta_a g\left(\Delta\beta^1\right)\sigma^2/\delta\mu^2. \qquad \square
$$

19

Proof of Proposition 3

We shall first show that the expression obtained for the optimal bias is a global maximum. Therefore, let us introduce the second order condition of the regulator's programme is given by:

$$g'' \left(\Delta \beta^2 \right) / \left[-g'(\Delta \beta^2) \right] < 2 \delta_a g \left(\Delta \beta^1 \right) / \delta \mu^2.$$

After a few combinations and given the assumption on the distribution of production shocks (Gaussian), the second order condition (16) becomes:

$$\left(\Delta \beta^2 \right) < g \left(\Delta \beta^1 \right) \frac{\delta_a \sigma^2}{\delta \mu^2} + \frac{\sigma}{\delta \mu^2} \sqrt{\delta_a^2 g \left(\Delta \beta^1 \right)^2 \sigma^2 + \delta^2 \mu^4}.$$

We shall now on show that the optimal bias $\left(\Delta \beta^2 \right)^*$ always respects this condition. Introducing (15) in (16), gives:

$$2 \delta_a g \left(\Delta \beta^1 \right) \sigma^2 / \delta \mu^2 < g \left(\Delta \beta^1 \right) \frac{\delta_a \sigma^2}{\delta \mu^2} + \frac{\sigma}{\delta \mu^2} \sqrt{\delta_a^2 g \left(\Delta \beta^1 \right)^2 \sigma^2 + \delta^2 \mu^4},$$

which after a few combinations becomes $\frac{1}{\sigma^2} \delta^2 \mu^4 > 0$ which is always true.

We now turn toward the marginal effects of the parameters on the optimal bias. Since all the parameters in expression (15) are positive the effects mentioned in Proposition 3 are straightforward. Moreover, the dynamic bias diminish with the first period static bias because the density function g reaches its unique peak at 0, when the initial bias increases $g \left(\Delta \beta^1 \right)$ decreases. $\square$

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