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Sequential problem choice and the reward system in Open Science

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Abstract

In this paper we present an original model of sequential problem choice within scientific communities. Disciplinary knowledge is accumulated in the form of a growing tree-like web of research areas. Knowledge production is sequential since the problems addressed generate new problems that may in turn be handled. This model allows us to study how the reward system in science influences the scientific community in stochastically selecting problems at each period. Long term evolution and generic features of the emerging disciplines as well as relative efficiency of problem selection are analyzed.

JEL classification: A12; C63; H40; O30

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1. Introduction

Nelson (1959) and Arrow (1962) first highlighted that the specific characteristics of knowledge considered as a public good result in a default in knowledge creation incentives. Consequently private investment in knowledge creation is below its optimal level. This very well known result appeared as a theoretical justification for public support of research which may (non-exclusively) be undertaken by funding a specific social institution, namely academia. In that respect, modern

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countries obviously support a network of public laboratories and academic researchers. After having focused on the social returns of public research, [1] economists have logically begun to address the issue of the internal organization of the academic institution.

Dasgupta and David (1994) have recently synthesized in an economic fashion the mertonian mechanisms at play within academia. According to Merton (1957), the functioning of the academic institution, he labels *Open Science*, relies on social norms [2] that generate a set of effective rules which stress a specific *reward system* in which *priority* is essential. The incentive mechanism at play may be sketched as follows. Peers collectively establish the validity and novelty of knowledge produced (peer review). The attribution of rewards is based on recognition by peers of the "moral property" on the piece of knowledge produced which increases the producer's reputation within the community ("credit"). Dasgupta and David (1994) highlighted that Open Science functioning has two fundamental and original economic properties that contribute to its efficiency. First of all, it avoids some of the asymmetric-informational problems that might otherwise arise between funding agencies and scientists in public procurement of advanced knowledge: scientists themselves are certainly the most able to carry out verification and evaluation operations in the peer-review like procedures. Secondly, since it is precisely the very action of disclosing knowledge which induces the reward (reputation or credit increase), the reward system thus creates simultaneous incentives both for knowledge creation and for its early disclosure and broad dissemination within the community. That is why this mode of knowledge production has been said to have very interesting efficiency properties (Arrow, 1987) and even to constitute a "first best solution" for the appropriability problem (Dasgupta and David, 1994) as it solves the dilemma between knowledge creation incentives and knowledge disclosure incentives (Stephan, 1996).[3]

Several modelling exercises have considered specific dimensions of the academic institution. Carmichael (1988) attempts to explain why does the tenure system exist: it is the only reliable employment contract that guaranties scholars will provide correct advises for employing high quality colleagues who might otherwise challenge their own positions. Lazear (1996) models the effects of several funding rules (e.g. weight more past efforts or the quality of the proposal, engage few big or many small awards, favor junior or senior researchers) on the incentives provided to scholars. Windrum and Birchenhall (1998) study the impact of the credibility based funding pattern on the evolution of a population of research units. Brock and Durlauf (1999) introduce a model of discrete choice between scientific theories when agents have an incentive to conform to the opinion of the community. Levin and Stephan (1991) propose a human capital model of knowledge production which fits the usual inverse-U shape of life-cycle scientific productivity. Carayol (2005) proposes a model of scientific competition in which overlapping generations of researchers compete at the different stages of their career while universities also simultaneously compete to hire the best scientists.

In this paper we focus on another dimension of academic organization, namely the sequential determination of research agendas within scientific communities and the subsequent disciplinary knowledge production. Our point of departure is that even though competition between scientists is clearly important (associated with "winner-takes-all" rules and "waiting and racing games"

[1] For a recent and complete survey see Salter and Martin (2001).

[2] Literally, Merton (1942) labelled such norms "institutional imperatives". Those norms are: "universalism, communism, disinterestedness, and organized skepticism".

[3] Of course, many problems still arise and it is not possible to derive from this statement that the decentralized allocation of research efforts induced by the specific reward system of science is *per se* optimal. This observation leaves room for an *Imperfect Economics of Science* to come (for a first investigation see Carayol, 2001).

issues, cf. Dasgupta and David, 1987; Reinganum, 1989), it is only second, while the first and most important decision a scientist has to take is the choice of which research area and which problem she or he will investigate. This issue is usually referred to in the sociology of science as the "problem of problem choice" (Merton, 1957; Zuckerman, 1978; Ziman, 1987). [4] As a matter of fact, a very consubstantial organizational trait of the Open Science is the significant freedom given to scholars in defining and selecting their own *research agendas*. More, the selection of good problems is far from being marginal from scholars' points of view in the academic competition: not all problems are the same in their eyes and in the ones of their peers.

The model introduced in this paper addresses the issue of the impact of the Open Science reward system on the allocation of attention of the community of scientists *ex ante*, and on the resulting evolution of disciplinary knowledge *ex post*.[5] Scientific disciplines are represented as growing tree-like webs of research areas that are the repository of accumulated knowledge. At each period, researchers allocate their attention responding to academic incentives. It leads to the improvement of knowledge in a given area or to the investigation of a new area. Our main results are that the process exhibits path dependency (David, 1985) especially disciplines that are more specialized have a higher chance to become even more specialized. We also find that there is a decline in the generation of new research areas over time which can be balanced by increasing the rewards for performing pioneering research. We also study how the outcoming disciplines are shaped through tuning the various typical incentives of the Open Science rewarding process. Finally, we propose a welfare criterion which assigns a given social surplus to each new problem addressed. We show and discuss how to balance academic incentives for improving the decentralized allocation of scholars' attention.

The paper is organized as follows. The next section discusses the issue of modelling problem choice and subsequent evolution of disciplinary knowledge. The technical presentation of the theoretical model is the purpose of the third section. The fourth section is dedicated to the study of the generic properties of the process, while the fifth section studies parameters effects on the dynamics and discusses the characteristics of the outcoming disciplines. The sixth section introduces a welfare criterion and analyzes how the reward system should be tuned for an efficient allocation of attention. The last section concludes.

2. Modelling problem choice in science

This section aims to ground our model of problem choice on what is known on problem choice in science. We first survey the literature on incentives provided to scholars for choosing problems in science and next expose, in a non-technical manner, the main features of our model of sequential selection of problems and of disciplinary knowledge expansion.

2.1. Problem choice in science

The issue of problem choice is complex and encompasses at least two sub-issues. First, how the selection of problems operates given their generic attributes of difficulty and expected returns?

[4] The history of this issue goes back to Peirce (1896) who provided the first formal model of the optimal allocation of research between projects characterized by different levels of utility and risk.

[5] A well known model of science growth is the one of Price de Solla (1963). For a recent simulation exercise see Gilbert (1997). Our approach is different since we explore problem choice (and thus individual incentives) in the context of a growing body of knowledge.

And second, what are the attributes of problems the resolution of which brings higher returns than others?

Among the factors that influence problem selection we may first acknowledge that scientists' choices over scientific problems are obviously a function of their chances of success. Polanyi (1962) argues that science is a self-organized institution for orienting scientists' attention and research efforts in a decentralized manner: an "invisible hand for ideas". The author claims that science is a well designed social institution mainly because it brings scholars' attention to the most easily solvable problems at each moment in time. The efficiency statement is of course questionable because the agents may under or over-invest their attention on problems due to the winner-takes-all nature of the reward system and because it relies essentially on a cost minimization device while the expected returns of solving problems should be considered. Merton and Merton (1989), who built an optimal control model in which researchers' efforts are dedicated to solving a given set of problems, simultaneously consider the intrinsic difficulty of problems, their relative importance and the intensity of rivalry between researchers.[6]

This brings us to the second sub-issue: why are some problems considered as more important than others in scholars' eyes (*ex ante*) in the eyes of their peers (*ex post*)? One would say a problem is an important one if it is expected to have significant returns for the one who solved it in terms of credit among peers. But then we shall document the attributes of problem the resolution of which brings more credit. According to the sociology of science, one important factor is the *novelty* of the issue addressed. Crane (1972) shows that, over a 25 years period evolution of a given emerging field of research, most of the main discoveries are found in the first 10 years while the scientific community is still quite small and little is found in the remaining period while the topic became considerably busy. Being first brings higher chance to pick the best and most important problems of a given domain.

But novelty is not sufficient *per se*. To be noticed, a contribution needs to be followed and to be acknowledged as a relevant and useful piece of knowledge. One obvious manner to do so consists in producing what Cohen et al. (1998) call "foundational knowledge", that is knowledge that opens significant opportunities of research for others. Mullins (1972) monograph on the Phage group and the subsequent development of molecular biology consistently supports the idea that pioneers long investigate the foundations of a new field before the crowd comes in. In a longitudinal study of research area investigation in radar meteor research, Gilbert (1977) shows that early investigators are also the most prolific scholars on each thematic area though they bore a higher risk to see their work not being widely noticed. Lemaine et al. (1976) underline that the thematic migration is also strongly motivated by the current state of the originating field which often offers fewer and fewer opportunities as compared to the destination one. In their quantitative study, Debackere and Rappa (1994) show that both dissatisfaction with the originating domain and positive signals on the destination field given by the successes of the early investigators are among the main factors that affect the choice of thematic mobility.

The monograph of Latour (1993) reports the strategies of a successful biologist and describes how he jumps from one issue to the other according to the expected rewards associated to each action. In the scientists' calculations leading to the choice of problems the scientist tries to gain larger *audience* by producing statements that cover a wider array of subproblems that are the

[6] Unfortunately, this paper which was initially supposed to appear (and was announced) in *Rationality and Society* in 1989 was withdrawn from publication at the time. The available version is still uncomplete and most results are still unavailable.

concern of the largest communities of researchers. To enlarge claims appears to be a key factor for being widely noticed and cited at the different stages of the "credit cycle" (Latour and Woolgar, 1979).

2.2. Non-technical presentation of the model

It follows from the previous subsection that problem choice should be analyzed in the dynamic context of a growing body of knowledge because the further developments of a discipline condition the *ex post* attribution of rewards associated to the production of a given piece of work.

More precisely, our model is designed to capture two features of the process of collective knowledge creation which are essential for analyzing scientific knowledge production within academic communities. (i) The problems to be addressed are neither fixed nor independent: rather our aim is to account for the idea that new problems are generated by previously created knowledge, that is by problems addressed in the past. This observation highlights the sequential nature of knowledge production. (ii) The rewards for solving problems are not exogenously specified: the allocation of research efforts to a set of handleable problems at any given period of time is derived from both the generic *motivations* of scientists and how they presently evaluate the recompenses for solving each problem defined according to the specific reward system in science above mentioned.

In order to take these two features into account we propose a model using graph theoretical principles: we assume that *disciplines* (a term by which we simply mean the *accumulated knowledge of a scientific discipline*) are designed as a *web of research areas* (i.e. a graph, the nodes of which designate research areas). The initial idea of scientific knowledge as a web of " theories" is to be found in Kuhn (1962).[7] The research areas are the unitary level of scientific knowledge organization. Each one is simply defined by its *location* in the graph and its *level of improvement*, that is the *number of problems* that have been addressed within that area (which can also be understood as the past accumulation of knowledge there). For the sake of simplicity, this web is assumed to be a *tree*-like graph. Thus we retain the classical, even if somewhat misleading, representation of scientific knowledge as a *tree of knowledge*.[8] Even if this simplification is far from being perfect, it makes it possible to clearly organize areas according to their level of specialization, through their geodesic distance to the root which is assumed to be the locus of the most general knowledge. The set of already existing research areas (nodes) and possible ones, their structures and their respective levels of improvement, together constitute the present state of the discipline. According to the sequential character of knowledge production highlighted in (i), the state of the system is assumed to directly give the set of attainable problems. At each period agents choose between the next problem at hand in an existing research area or exploring a new research area. The latter is modelled by introducing the opportunity that any already improved research area leads to the potential creation of a new research area (which one could view as a new *leaf* of the tree) by solving its first problem.

In the context of our model, the remaining (ii) issue becomes: how do agents strategically allocate their *research efforts* over the set of handleable problems? In order to capture that, we introduce a specific *reward function* determining how agents associate at a given moment in time an expected reward to each attainable problem. Incentives to perform research on research

[7] Even if very different in its conception, see also Weitzman (1998) for one of the first contributions 'discretizing' knowledge.

[8] See Machlup (1982) for a history of this notion which dates back to Lull, Bacon and Comte. Some first elements can also be found in Cournot (1861) where the idea of a *branching* evolution of knowledge in the sciences and industry is introduced. See also Ziman (1994) for a rationale about its applicability to the usual scientific classification systems.

areas are assumed to be a function of three variables that have been highlighted by the empirical literature on problem choice in science we highlighted in the previous subsection and which can be simply computed in the tree-like structure of disciplines. The first variable indicates how much the research associated with a given problem is *pioneering*. It is obtained by computing the number of problems already addressed so far in the research areas considered. The second variable is the relative *difficulty* of research that we proxy by the level of *generality* of the area (its inverse, *specialization*, is computed as the distance of the research area to the most general area, the root). The third variable relates to the *audience* of research areas. It is to be understood as the expected volume of further research that may refer to a scientific contribution appearing in a given area. Since scientific papers tend to cite papers belonging to the same or to connected areas, we will assume that the audience of a research area increases linearly with the size of the associated sub-field. Here lies a decisive support for the tree-like representation which allows us to simply define the size of an area sub-field as the size of its associated *sub-tree*. Since the aim of the model is to focus on incentives (problem choice) the tree-like structure captures most of the dynamics principles governing research effort allocations that are highlighted in (ii).[9]

Lastly, what we need is an *instantaneous aggregation procedure*, so as to infer collective outcomes from individual choices (Kirman, 1992): we introduce a simple probabilistic rule which provides the instantaneous allocation of efforts over the research areas given the relative incentives, which in turn generate the effective ‘arrival’ of knowledge at each period of time. This rule is also designed to capture the idea that the concentration of competitive pressure on the most rewarding areas may also influence the dynamics. [10] Altogether, one obtains a stochastic process that inter-temporally aggregates scientists’ efforts and leads to the growth of the discipline (the tree). This original process is denoted by $\{T^t | t = 1, \ldots, \tau\}$. Since it is based on dynamic growing trees, this process has some common features with the *Bienaymé-Galton–Watson branching process* in applied mathematics (Kulkarni, 1995), the literature on tree indexed processes in Probability (Lyons and Peres, 2002) and the *avalanches* literature in Physics (for an application to economics, see Plouraboue et al., 1998). Since this process leads to complex dynamics, its properties are analyzed using standard *Monte-Carlo experiments*.

3. The model

This section is dedicated to the formal presentation of our model. We first define the disciplines, seen as more or less improved research areas organized as nodes in a tree-like web. Next we discuss and show how these disciplines generate a set of problems that can be handled given the present state of knowledge. Thirdly, we introduce an expected reward function which provides the incentives associated with solving each of the available problems. Finally, we present the probabilistic function implementing scientists’ choices at each period and thus the advances of scientific knowledge.

3.1. Scientific disciplines

At each period t of the discrete time, let a scientific discipline be described as an undirected graph G^t. A graph is formally defined as a double set: $G^t \equiv \{V^t, E^t\}$, where V^t is the set of the nodes (i) (vertices) of the graph and E^t is the set of the edges (ij) of the graph (for a general

[9] For a model of unspecified growing knowledge graphs see Carayol (2000).

[10] This idea was introduced in Merton (1957), Merton and Lewis (1971), surveyed by Zuckerman (1978).

presentation of graph theory see Diestel, 1997). Any edge $(ij) \in E^t$ is a link between the two nodes (i) and (j) belonging to the set of nodes: $(i), (j) \in V^t$.

We represent scientific disciplines as ‘knowledge trees’ where nodes designate different but interdependent research areas. Substantially, the root corresponds to the most general research area, while ‘lower’ nodes correspond to more specialized areas of knowledge. Since we consider trees, the extra properties follow:

- the graph is connected, i.e. there is a path relating each pair of nodes of the graph: $\forall(i)$ and $(j) \in V^t$, $\exists$ a path $\{(i, l), (l, u), \ldots, (k, j)\} \subset E^t$;
- the graph is minimally connected: there is only one path between any two nodes of the graph (there is no cycle in the graph). As a consequence, we have a direct correspondence between the cardinal of V^t and the cardinal of E^t : $\#E^t = \#V^t - 1$.

Moreover, a tree is also a planar graph: it can always be represented on a plan without any crossing between edges. Since there is one and only one path between any two nodes, a tree has also an unambiguous geodesic distance which is a counting function of the number of edges of the path connecting two nodes. Thus $d(i, j)$, the distance between the nodes (i) and (j), is given by: $d(i, j) \equiv \#\{(i, l), (l, y), \ldots, (k, j)\}$. The root of the graph is a specific node denoted (1). The distance to the root $d(i, 1)$ can simply be denoted d_i. Thus $d_1 = 0$. The root is assumed to represent the research area with the highest possible level of generality. The distance to the root then expresses the level of specialization of each research area. Let $G_j^t = \{V_j^t, E_j^t\}$ denote the sub-tree of $G^t (G_j^t \subset G^t)$ the root of which is node $(j).G_j^t$ is said to be a sub-field of the discipline G^t associated to area (j). Thus, we have by definition $G_1^t \equiv G^t$. The size of the sub-discipline G_j^t is given by s_j^t, the number of research areas of G_j^t: $s_j^t \equiv \#V_j^t$. The tree structure (having also specified that (1) is the root of the tree) allows us to clearly define the ‘father’ operator, $f_{G^t}(\cdot)$ which gives the ‘father’ of any of the nodes of V^{t*}.

Finally, let us define the vector $\Phi^t = (\phi_i^t)_{i \in V^t}$ which assigns to each node of the graph $(i) \in V^t$, a non-null integer $(\phi_i^t) \in \mathbb{N}^*$, $\forall i \in V^t$ denoting the robustness of knowledge (or its improvement level) attained within the research area (i) (thus $\Phi^t \in (\mathbb{N}^*)^{\#V^t}$). It is defined as the number of problems addressed in the research area considered.

Together, the set of nodes, the set of edges, and the improvement levels define the state of the discipline (the system) at each period, i.e. the knowledge accumulated so far and its structure. It is denoted by $T^t \equiv (V^t, E^t, \Phi^t)$

3.2. Available research agendas and problem generation

Let us now define the set of attainable problems given the present development of the discipline, i.e. the set of potential research agendas that the present state of the system allows us to tackle. As argued above, one cannot reasonably assume that the set of problems is fixed and *ex ante* specified, but rather generated and even shaped by the advance of knowledge. In order to capture that feature, we consider that scientists can either (i) improve knowledge in some already existing research area, or (ii) create a new research area from an already existing one. In other words, the problems that may be addressed are either the next problem at hand in already improved research areas or the first problem to be addressed of a research area connected to any already improved one.

In this last respect, we further consider that a new 'virtual' node is associated with each existing one: that is, we consider that scientists are able to create a new and more specialized research area 'from' any existing one. Let W^t be this set of nodes that can be created at period t: each (j) in W^t is simply a *leaf* from each node $(i) \in V^t$, the improvement level of which is still null: $\forall (j) \in W^t : \phi_j^t = 0$. Obviously, we have: $\#V^t = \#W^t$ and $V^t \cap W^t = \emptyset$.

Let us now define the set O^t such as: $O^t = V^t \cup W^t$. It is the set of opportunities, that is the set of research areas that contain a handleable problem at date t (whether the research areas have already been improved or not). We also need to define the graph that covers the opportunities, that is $\Lambda^t \equiv \{O^t, F^t\}$ such that its set of edges F^t is the union of the set of edges E^t (of graph G^t) and of the set of edges say I^t that is binding each node of V^t to one distinct node of $W^t : F^t \equiv E^t \cup I^t$.

We assume that the state of the system simply evolves by selecting one node among the set of opportunities at each period. The selected node at period t is denoted by $(o^t) \in O^t$. Formally, when a node is selected, the two following events transform the state of the system:

$$\text{(i)} \quad \begin{cases} \text{if } (o^t) = (i) \in V^t \text{ then } V^{t+1} = V^t; W^{t+1} = W^t \\ \text{if } (o^t) = (l) \in W^t \text{ then} \\ V^{t+1} = V^t \oplus (l); W^{t+1} = W^t \ominus (lt) \oplus (k) \oplus (h) \\ E^{t+1} = E^{t+1} \oplus (ul); I^{t+1} = I^t \ominus (ul) \oplus (lk) \oplus (uh) \end{cases} \tag{1a}$$

$$\text{(ii)} \quad \phi_{o^t}^{t+1} = \phi_{o^t}^t + 1. \tag{1b}$$

The symbol $\ominus$ simply indicates that the node considered is removed from the set. By $\oplus$, it is meant that the element is added to the set. Eq. (1a) says that if the chosen research area is an already improved one ($(i) \in V^t$), the set of nodes and potential ones are left unchanged. If the chosen node is a new one ($(l) \in W^t$) it is then added to the set of already improved nodes, while it is withdrawn from the set of potential ones. In the meantime, two new elements are added to that set. The first one (k) is the new research area that may now be attained because of the initial improvement of its father (l) on the set of opportunities O^t. The second one, (h), replaces (l) within the set of potential areas. This feature of the model conveys an implicit assumption of the model, namely that there is no scarcity in potential research areas. Each new investigation solving a problem generates new questions both inside its research area and in new areas. Eq. (1b) states that the level of improvement of the chosen research area is improved by one unitary increment. One can thus verify that the level of improvement of each node is equal to the sum of all the problems addressed in the past within that research area: $\phi_i^t = \sum_{\tau=1,\ldots,t} 1\{i = o^\tau\}$ with $1\{\cdot\}$ the indicator function.

An example of such a process is represented in Fig. 1 when starting with only one node at period 1 (the root). The numbers on each node are the levels of improvement ϕ_i^t (number of problems addressed there), and the dotted lines represent possible areas that may be created i.e. virtual nodes. When one denotes each node by an integer which is increased by a unitary increment at each arrival, then the evolution of the tree described in Fig. 1 can be formally written as in Table 1.

3.3. Incentives and motivations for problem choice

We now tackle the issue of defining the incentives that may lead researchers to select projects among the available ones. For that purpose we introduce a reward function $\omega(\cdot, \cdot, \cdot)$ that associates

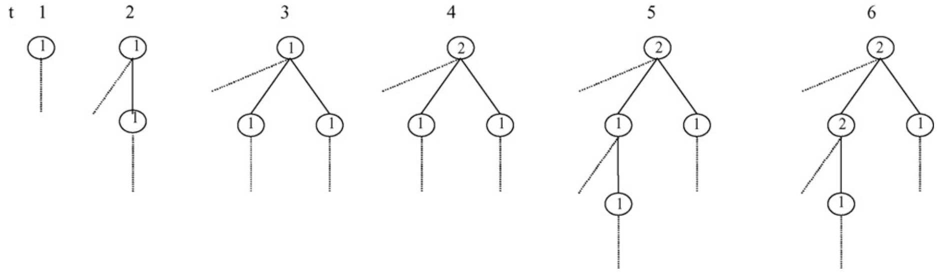

Fig. 1. An example for the evolution of a knowledge tree over the first six periods of time.

Table 1
The evolution of the tree-like graph described in Fig. 1

t	V_t	E_t	Φ_t	W^t	O_t	o_t
1	{1}	Ø	(1)	{2}	{1,2}	{2}
2	{1,2}	{(1,2)}	(1,1)	{3,4}	{1,2,3,4}	{3}
3	{1,2,3}	{(1,2),(1,3)}	(1,1,1)	{4,5,6}	{1,2,3,4,5,6}	{1}
4	{1,2,3}	{(1,2),(1,3)}	(2,1,1)	{4,5,6}	{1,2,3,4,5,6}	{4}
5	{1,2,3,4}	{(1,2),(1,3),(3,4)}	(2,1,1,1)	{5,6,7,8}	{1,2,3,4,5,6,7,8}	{2}
6	{1,2,3,4}	{(1,2),(1,3),(3,4)}	(2,2,1,1)	{5,6,7,8}	{1,2,3,4,5,6,7,8}	–

a given expected reward ω_o^t to each possible problem $(o) \in O^t$ at any given period of time t. This expected reward is assumed to be given by

$$\omega_o^t = \omega(\phi_o^t, d_o, s_o^t), \tag{2}$$

with ϕ_o^t which has been defined as the level of robustness of the research area, i.e. the sum of past accumulation of knowledge on (o). When (o) corresponds to a new research area, then still no knowledge has been accumulated and thus $\phi_o^t = 0$. d_o which is the distance to the root (let us recall that $d_o \equiv d(o, 1)$) indicates the level of specialization of the research area considered (o). When d_o is low, the research area is general while if d_o is high, (o) tends to be specialized.[11] s_o^t is the size of the sub-discipline G_o^t associated with the research area (o). If (o) is a new research area then obviously, $s_o^t = 0$. s_o^t is a proxy for the "audience" associated with research areas (o).

We propose a simple Cobb Douglas specification for (2) as follows:

$$\omega_o^t = (1 + \phi_o^t)^{\gamma}(1 + d_o)^{\lambda}(1 + s_o^t)^{\delta}, \tag{3}$$

where $(1 + \phi_o^t)^{\gamma}$ (with γ such that $\gamma < 0$) stands for a measure of the relative rewarding of problems in academic publications depending on their novelty. It is a well known property of the reward system in science to reward more pioneering research: papers appearing in a given area are likely to acknowledge the first contributors. Most important scientific prices and awards such as the Nobel Prize are also likely to reward pioneering research. Therefore the less original the next problem addressed (the higher ϕ_o^t) the lower the associated reputation gain (because $\gamma \leq 0$). The reward of the nth contribution to a given research area is only a fraction of the first one tuned by γ: when γ is small, $(1 + \phi_o^t)^{\gamma}$ tends to highly decrease when ϕ_o^t increases (especially when ϕ_o^t is still small). On the contrary, when γ is close to 0, the decreasing slope is lower. Thus when

[11] It is a specific and constant attribute of the research area considered, independent of the time period considered. Thus the time period superscripts have been removed.

γ is close to 0, the first problems addressed within a given research area tend to be relatively as rewarded as the later ones, while when γ is much lower than 0, pioneers will be much more rewarded than later contributors.

The second component of expression (3), namely $(1 + d_o)^\lambda$, stands here for a measure of the relative rewarding for scientists for producing knowledge in a given area depending on its generality/specialization. This parameter is clearly influenced by the role of experiments in a discipline, but mainly accounts here for the difficulty to perform *valid* and therefore rewarding research at a high level of generality both because of its inherent complexity and because access to high-level equipment is needed and difficult to secure. Parameter λ is therefore assumed to be positive ($\lambda \geq 0$), which implies that the *expected* reward is higher for more specialized areas. This feature may appear counter-intuitive at first glance, [12] but the idea behind this assumption is that the more general a research area, the more difficult it is to perform valid research there: to put it differently, too difficult research areas correspond to lower expected rewards.[13] λ controls the relative preference/rewarding for specialized research areas: when it is close to 0, specialized and general areas are equally difficult and efforts are equally rewarded.

The last component of the expected reward function $(1 + s_o^t)^\delta$ measures the impact of the audience of research areas on the expected rewards. When the associated sub-discipline is big, i.e. is composed of a large number of research areas, then the research results that may appear in such a research area are more likely to be frequently cited. The parameter δ (such as $\delta \geq 0$) expresses the relative importance of that factor in the reward system. It may be due to the strength of the citation system or even the 'vertical' integration of the discipline that lead applied areas to be aware and often use general statements. When $\delta \to 0$, the size of the sub-tree of area (o) tends to have no more any effect on the expected rewards ω_o^t. The more δ increases, the more the audience $(1 + s_o^t)^\delta$ becomes an important part of the rewards (s_o^t becomes critical).

3.4. Sequential discrete choices and the stochastic process

In the preceding subsection we have stressed a payoff function which gives the incentives for research agenda determination. Let us now turn toward defining how problems are effectively selected among all opportunities on the basis of such incentives. At each period, each opportunity (o) is assumed to be selected and addressed with the following probability:

$$p_o^t \equiv \frac{(\omega_o^t)^\alpha}{\sum_{j \in O^t} (\omega_j^t)^\alpha}, \tag{4}$$

with $\sum_{o \in O^t} p_o^t = 1$, $\forall t$ and α ($\alpha > 0$). α stands for the concentration of scientists' attention on the most rewarding areas for a given allocation of incentives over the handleable problems. For instance, when $\alpha < 1$, the less rewarding research areas tend to be selected 'more frequently'. On the contrary, when $\alpha > 1$, the more rewarding areas tend to be selected more

[12] Usually more general statements are more rewarded not because they are more general but because they are of interest for a wider range of further research. In the model this corresponds to a larger audience (s_o^t) and this is not *per se* in contradiction with more specialized areas being more rewarded. It is the tree representation that allows us to capture simultaneously both types of incentives.

[13] A later version of the model presented here should include true stochastic rewards, for instance by attaching a probability to each research area which would drive the *ex post* success of efforts in problem solving for each. Then heterogeneous agents could consider choosing a more difficult area when their risk aversion would be low.

than proportionally to their expected rewards, while less rewarding areas tend to be chosen less than proportionally. As $\alpha \to \infty$ the node associated with the highest reward is almost surely chosen, and the system becomes 'quasi-deterministic'. On the contrary, when $\alpha \to 0$, each possible opportunity is chosen with the same unique probability: $p_o^t \to p^t$, $\forall t$ and $(o) \in O^t$.

Introducing Eq. (3) in Eq. (4) gives us the probability that any research area (whether it is an existing one or a potentially created one) is improved at period t:

$$p_o^t = \frac{((1+\phi_o^t)^\alpha (1+d_o)^\alpha (1+s_o^t)^\delta)^\alpha}{\sum_{j\in O^t}((1+\phi_j^t)^\gamma (1+d_j)^\lambda (1+s_j^t)^\delta)^\alpha}. \tag{5}$$

This closes the description of the stochastic process of knowledge generation within disciplines. Formally, we obtain a stochastic discrete time infinite space state process that we denote by $\{T^t | t = 1, \ldots, \tau\}$ or conversely $\{(V^t, E^t, \Phi^t) | t = 1, \ldots, \tau\}$ since it describes the evolution through time of the two sets V^t, and E^t, and the vector Φ^t. Fig. 2 shows three dynamically grown trees that have resulted from different numerical experiments realized with such a stochastic process.

Together, the parameters $(\lambda, \delta, \gamma, \alpha)$ which appear in Eq. (5) are said to characterize an academic community and its associated reward system. α stands for the problem choice practice given the incentives. λ, δ, and γ give the relative weighting of the various types of incentives whether this weighting comes from the motivations of the agents or from the effective rewarding. In practice both may be very close since academic communities are, to a large extent, self-organized in the sense that agents are both the rewarded and the rewarding agents.

4. Generic properties of the process

Once the model has been presented, we first turn to an exploration of the generic properties of the system, that is the behavior of the dynamic process through time and its limit behavior, while the characteristics of the drawn trees depending on parameters values are described in the next section. As it has been said above, the system $\{T^t | t = 1, \ldots, \tau\}$ is a quite complex one which naturally leads to complex dynamics. To make that point clear, let us consider the following. From Eq. (5), it can easily be demonstrated that: $\forall t, \forall i \in O^t : (\partial p_i/\partial d_i) > 0; (\partial p_i/\partial s_i) > 0; (\partial p_i/\partial \phi_i) < 0$; under the assumptions made previously about the parameters value spaces ($\gamma < 0, \delta > 0; \lambda > 0$ and with $\alpha > 0$). This implies that, all things being equal, more specialized research areas, new research areas and research areas with larger audiences are more attractive to scientists' choices. The problem is that these variables are dynamically correlated because the specialization and the audience variables are often negatively correlated and because the robustness levels (ϕ_i) act as a 'crowding out' variable which tends to re-allocate incentives through time over the population of research areas (while it also regulates partly the creation of new ones). In order to analyze the complex behavior of the system, we mostly rely on Monte-Carlo simulation experiments. All experiments that will be presented in the rest of the paper start with a tree reduced to its root at period one ($V^1 = \{(1)\}; E^1 = \emptyset; \Phi^1 = (1)$).

Among the various features of the system's behavior that one may wish to consider, we are particularly interested in the evolution of the two following ones: the generation of research areas (versus the improvement of existing ones), and the specialization of scientific knowledge (that is how far from the root the problems addressed are located). The two sub-sections below tackle these issues successively.

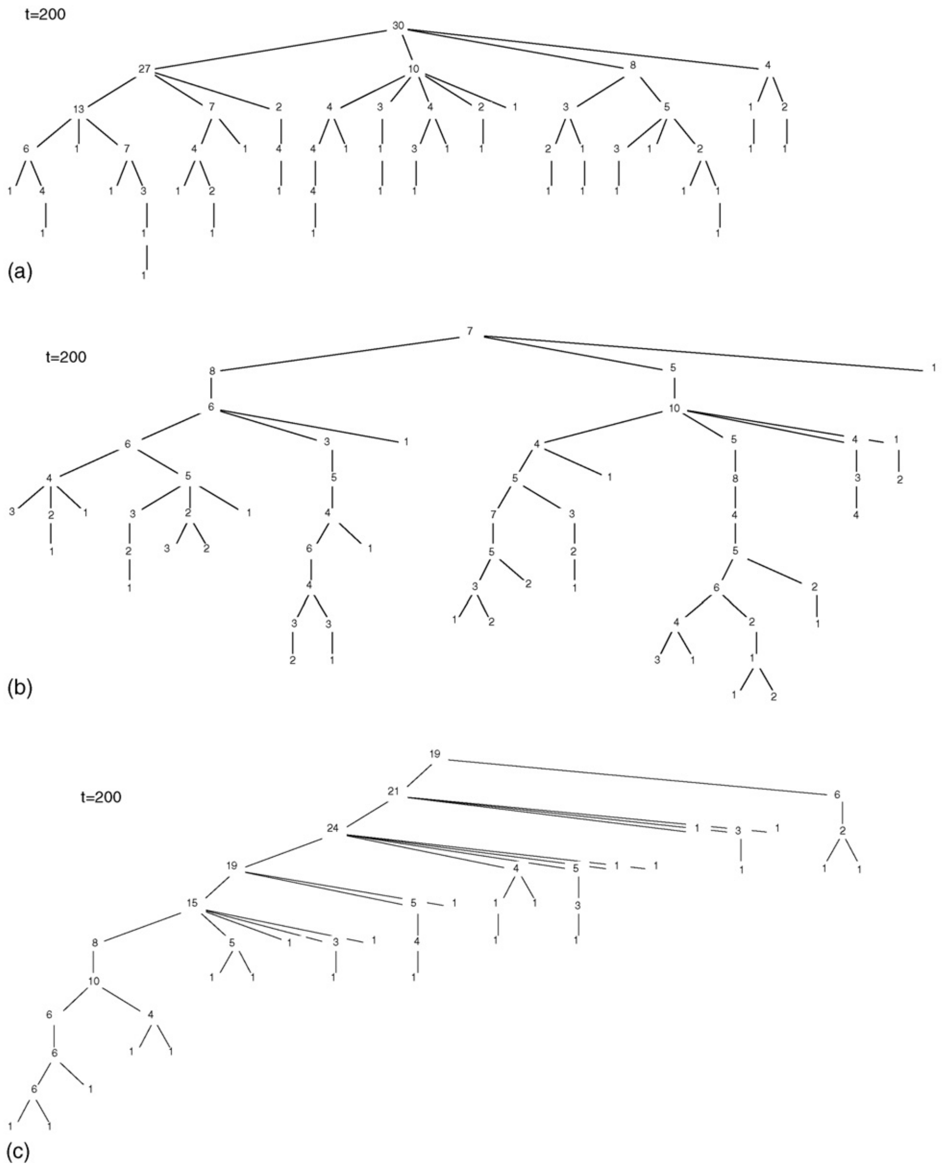

Fig. 2. Typical grown trees (200 periods).

4.1. The inexorable decline of research areas generation

To analyze collective outcomes, it is useful to define some aggregate measures. Let us define the *creation index* as the number of nodes created over the number of possible node creations or else the number of periods, i.e.

$$\varsigma^t \equiv \frac{1}{t} \# V^t. \qquad (6)$$

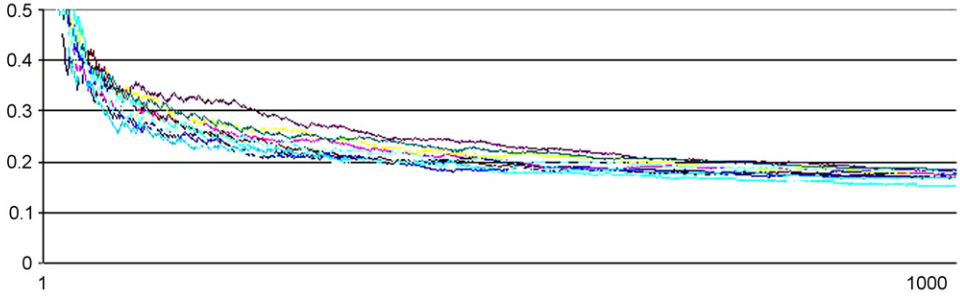

Fig. 3. Time series of the creation index ς^t over 1000 periods. Ten identical runs with: $\delta = \lambda = -\gamma = 1$, $\alpha = 4$.

Obviously, this index is such that $0 \leq \varsigma^t \leq 1$, because there is at most one creation per period. ς^t may also be called the *exploration index* because it captures the past ability of the community to explore new scientific paths.

One has to see that there is a real trade-off between creation/exploration and robustness, because any effort which is not dedicated to the creation of a research area is dedicated to the improvement of an existing one. Thus the average improvement of research areas given by: $\bar{\phi}^t \equiv \frac{1}{\#V^t} \sum_{(i)\in V^t} \phi_i^t$; is the inverse of the creation index: $\bar{\phi}^t = 1/\varsigma^t$; just because we have: $\forall t$, $\sum_{(i)\in V^t} \phi_i^t = t$.

Fig. 3 presents 10 identical runs of the dynamic process, recording at each period the creation index ς^t, the parameters being arbitrarily fixed. Many other numerical experiments with different values of the parameters have been run, all showing that *creation decreases with time*. We have also run experiments over very long periods showing that the creation index was still decreasing after 10,000 periods. A unique but important exception arises when $\alpha = 0$, that is when the evolution probabilities are equal at each period. In such a situation, it can easily be demonstrated that the creation probability is equal to 1/2 at any period.

This general result, is a simple but not so obvious consequence of the fact that knowledge improvements are rewarded by the attention of other scientists, often measured (even if very imperfectly) by counting citations received: the more a scientific discipline grows, the higher the incentives for improving knowledge in existing research areas just because their audience (sub-tree size) is relatively increasing as compared to incentives for exploring new paths.

4.2. General versus specialized disciplines: a path-dependent outcome

Let us define the average generality of knowledge index, as the sum of the robustness levels of all research areas, weighted by their level of generality (the inverse of the specialization level: $1/d_i$), over the sum of all the robustness levels, as follows:

$$\tilde{\phi}^t \equiv \frac{\sum_{(i)\in V^t}(\phi_i^t/d_i)}{\sum_{(i)\in V^t}\phi_i^t}. \tag{7}$$

This index gives the average generality of each problem addressed so far, which we can say is the average level of generality of knowledge within the discipline considered. Like the creation index, this index is such that: $0 \leq \tilde{\phi}^t \leq 1$, with $\tilde{\phi}^t$ close to 1 (to 0) when problems addressed have been highly general (specialized).

Fig. 4 presents 10 identical numerical runs of the process recording at each period the evolution of the knowledge specialization through the index $\tilde{\phi}^t$, all parameters being fixed. It

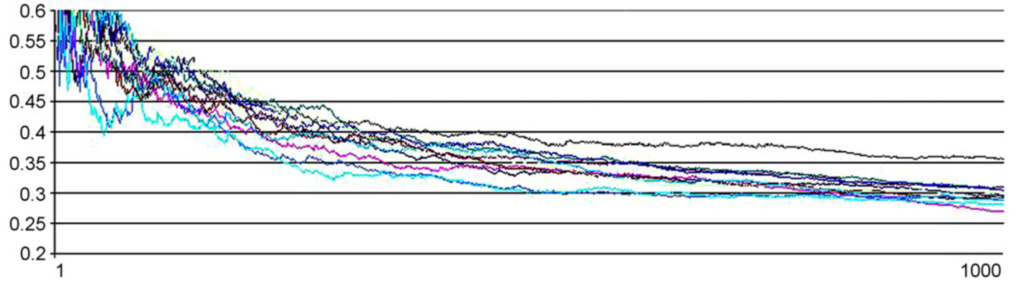

Fig. 4. Time series for the specialization of knowledge $\tilde{\phi}^t$. Ten numerical experiments realized over 1000 periods, with parameters fixed as follows: $\delta = \lambda = \alpha = 1$, $\gamma = -1$.

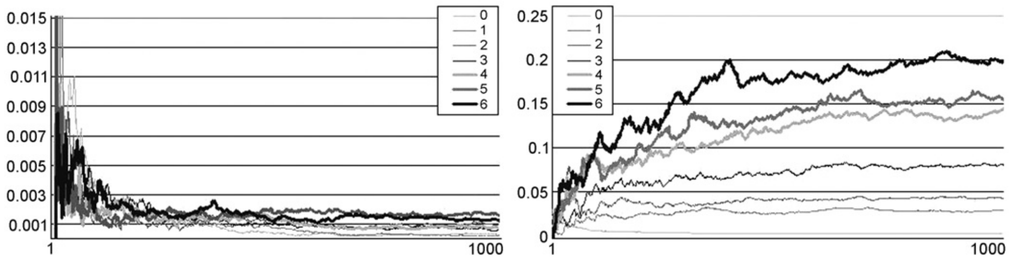

Fig. 5. Evolution, over 1000 periods, of the variation coefficient of both the creation index (graph a) and the generality index (graph b) from 10 identical runs of the process, computed for different values of α ($\alpha = 0, 1, \ldots, 6$). Numerical experiments realized with fixed values of the other parameters: $\delta = \lambda = -\gamma = 1$.

shows startingly that this is decreasing through time. This feature is robust to modifications of the parameters values. The explanation for such a result is quite obvious because, in the model, it is precisely the past exploration behaviors at specialized levels that allow agents to address even more specialized problems.

The other insight that time series may provide has to do with the path dependency property, which in dynamic stochastic systems theory simply means that the transitory states of the system (state variables) determine its limit behavior. Practically, if we find that there are increasing differences between identical runs of the system (same initial states and same parameters values), path dependency is said to occur. If differences tend to diminish, the time series tend towards a common and unique limit, thus the system is simply auto-regressive and does not exhibit the path dependency property.

To know which phenomenon occurs, we computed and recorded the variation coefficient (variance/mean) of the generality index $\tilde{\phi}^t$ of 10 identical runs of the process. As the parameter α is likely to be critical in such a respect we did the experiment for different values of α. In order to see potential qualitative differences with the creation index, we computed the variation coefficient for creation index ς^t. The results obtained for both ς^t and $\tilde{\phi}^t$ are presented in the graphs a and b of Fig. 5.

As one can observe in comparing graphs a and b of Fig. 5, one gets very contrasted results between the path dependency of the system while focusing either on its creation behavior or on its generality. [14] The variation coefficient of the creation index is clearly very small and decreasing

[14] Recalling that both indexes are comprised between zero and one.

toward zero as time goes (whatever the value of its parameter α). Thus clearly, the system does not exhibit a path dependency property when looking at the creation of research areas. The opposite occurs when one looks at the generality index. Indeed, one can observe in graph b that the variation coefficient of $\tilde{\phi}^t$ increases with time, the slope being positively influenced by parameter α. Thus the initial events – the first choices of where to address problems – can durably influence knowledge production. As a consequence, the dedication of scientific disciplines toward applied or more general knowledge can be influenced not only by their endogenous characteristics but also by the very history of disciplines and the path they took at the beginning. Early pioneers of new scientific disciplines are therefore responsible for shaping them and giving them a more or less 'applied' turn.

5. Incentives, motivations and the outcoming disciplines: parameters effects study

Now that the main generic dynamic properties of the system are known, we wonder how various combinations of parameters values (which stress the effective rewarding of problem selection) impact on the outcoming disciplines analyzed mainly by computing the two indexes defined in Eqs. (6) and (7) and discuss the generic features of the typical disciplines generated by opposed values of parameters.

5.1. Parameters effects study

To get clearer results, we present several simulations according to the following protocol. First, set $\alpha = 1$. Then each time one of the three parameters $(\delta, \lambda, \gamma)$ is fixed while the two others vary. For each couple of values of the two tuned parameters, we compute the values of the indices obtained after 1000 periods. From now on, we write ς° for ς^{1000} and $\tilde{\phi}^{\circ}$ for $\tilde{\phi}^{1000}$. Every single point in the graphs presented here (Fig. 6) then corresponds to the values of one index computed for a singular grown tree (thus 400 trees were generated for each simulation experiment). These 2×2 simulation process is used because it allows us to assess the robustness of the parameter modification effects for various values of the other parameters. For controlling purposes, the same experiments have been reproduced for different values of the fixed parameters. Moreover, the whole experiment has been reproduced with different values of α. As a first global result, we found that when α increases the slopes in the indexes due to modifications of the other parameters become steeper. Thus the effects discussed below are valid under different values of the community's sensitivity to the academic incentives tuned through parameter α. Table 2 summarizes the main results obtained.

Both a higher δ and a γ closer to 0[15] decrease creation: looking for citation rents, academic scientists tend to prefer improving knowledge within existing areas rather than exploring new ones, both when the size of the audience (an important sub-tree) counts much and when rewards for performing pioneering research are lower. Higher δ tends to favor research performed at a high level of generality because of the following: research areas which are more general are often (but not systematically) the ones that have the strongest audiences, i.e. research performed there is likely to be cited by research performed in more specialized but connected areas. Thus, general areas become naturally more attractive on average. Similarly since γ regulates the incentives to dedicate research efforts toward more or less improved research areas, when it increases it diminishes the creation of research areas and prevents the discipline from achieving an important

[15] Recalling that γ is negative, thus "γ increases" is similar to "γ becomes closer to zero".

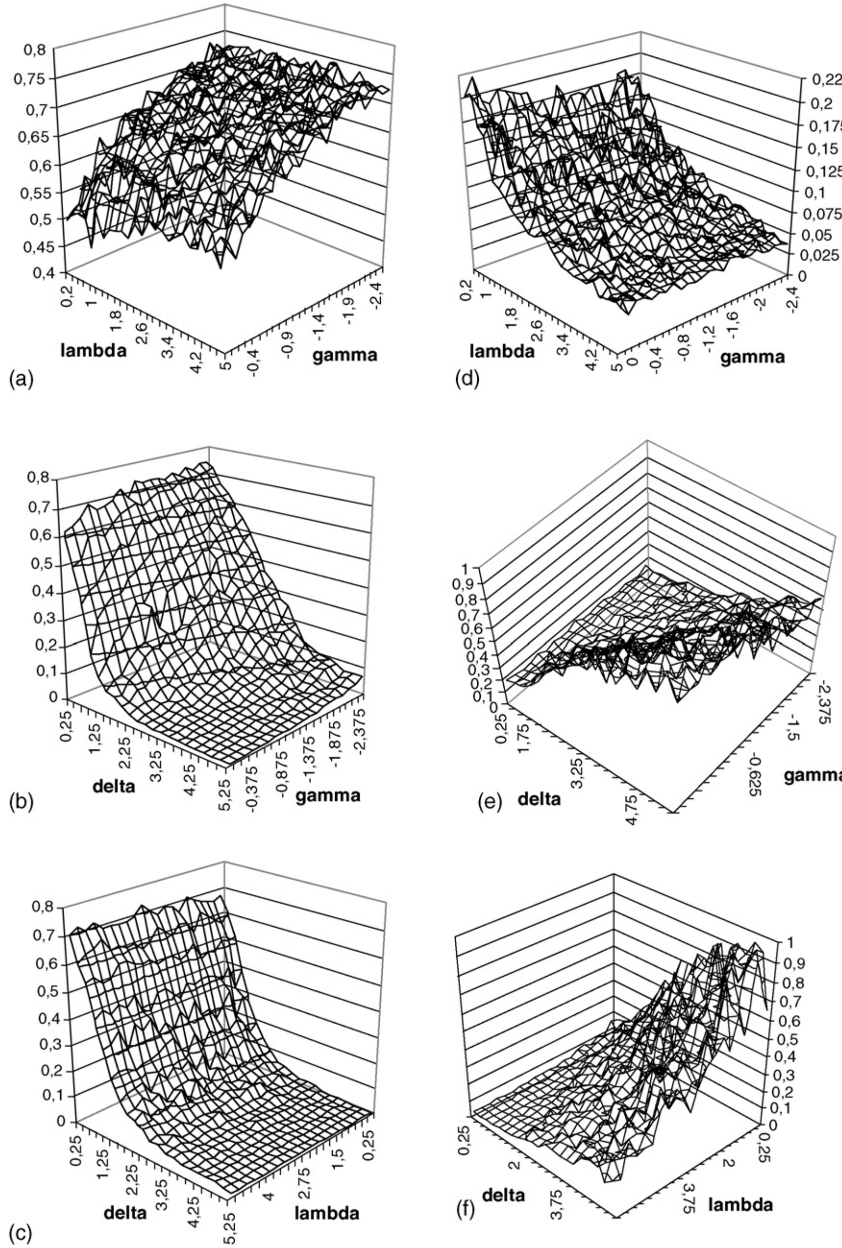

Fig. 6. Creation index ς° (left: graphs a–c) and generality index $\tilde{\phi}^{\circ}$ (right: graphs d–f). Graphs a and d: λ and γ vary, $\delta = 1$. Graphs b and e: δ and γ vary, $\lambda = 1$. Graphs c and f: δ and λ vary, $\gamma = -1$. In all experiments, results are observed after 1000 periods and with $\alpha = 1$.

Table 2
Effects of the parameters (δ, γ, λ) on the indexes (ς°, $\tilde{\phi}^\circ$)

	δ	γ	λ
ς°	$\searrow$	$\searrow$	$\longrightarrow$
$\tilde{\phi}^\circ$	$\nearrow$	$\nearrow$	$\searrow$

A positive effect is denoted by $\nearrow$, a negative by $\searrow$, and no effects by $\longrightarrow$.

level of specialization. This is derived from the fact that the model assumes specialization results from successive downward exploration. Thus some rather general disciplines may in fact be ‘under-developed’ trees, therefore lower specialization may be, under certain circumstances, just a consequence of a lack of exploration behaviors. Finally, higher λ increases the attractivity of more specialized research areas, and therefore the average distance to the root.

5.2. *How motivations and incentives lead to various outcoming disciplines*

We have just seen how the features of the outcoming disciplines are influenced by each parameter. We now take the opposite point of view, by taking polar forms of the discipline-trees and we wonder how they may be generated. Furthermore: what is their probability of occurrence because of reinforcing (or opposed) effects of the parameters? And finally, are these forms likely to be stable over time considering the generic properties of the process highlighted in the previous section?

Let us first define the four *polar forms* of disciplines coming from opposite values of the two indexes: the ‘star’ form is obtained when the discipline is both highly general and research areas are highly improved (high $\tilde{\phi}^\circ$, low ς°), a ‘well’ form is said to arise when the discipline is highly specialized and counts only few research areas (low $\tilde{\phi}^\circ$, low ς°), the ‘flake’ form comes from many creations at a quite general level (high $\tilde{\phi}^\circ$, high ς°), and the ‘rake’ form stands for both high specialization and numerous creations (low $\tilde{\phi}^\circ$, high ς°). These configurations are presented in Table 3 below.

We have seen that δ and γ play systematically in an opposite manner. Thus, on the one hand, they can counterbalance each other, in the sense that the parameter values which lead to specialized (general) disciplines are the same that prevent their areas from being much improved (numerous). That makes both the ‘star’ and the ‘rake’ forms quite unlikely to emerge since the specific traits of each form are generated by opposed values of the parameters. On the other hand, they may also reinforce one another. That leads to the idea that the ‘flake’ and the ‘well’ forms are quite probable ones: the factors that orient the disciplines towards being specialized (general) tend also to orient the research efforts toward being less (more) exploratory.

Nevertheless, as we have seen before, the creation of new research areas is likely to diminish through time: thus the ‘flake’ forms might be seen as a transitory state of the ‘star’ that may emerge in the long run: once the community has generated a certain number of research areas, the relative incentives for improving research areas may overbalance the exploration ones, and thus,

Table 3
The polar forms of the scientific disciplines

	High $\tilde{\phi}^\circ$	Low$\tilde{\phi}^\circ$
Low ς°	‘Star’	‘Well’
High ς°	‘Flake’	‘Rake’

among the existing areas, the general ones which have a larger audience become more attractive. As a matter of fact, the ‘star’ form has much in common with a disciplines such as Physics, which is an old and fully integrated discipline, which has explored many specific and applied areas and where the most theoretical and general discoveries may benefit to a large audience dispersed in many specialized research areas.

At the opposite corner, the ‘well’ form seems to be a quite interesting and robust typical shape of scientific disciplines since specialization is a path dependent outcome and exploration tends to decrease. Such disciplines emerge because of weak incentives for performing pioneering research and weak rewarding of research due to relative audience. The latter may come from scientific communities that would not (or weakly) take citations into account in the evaluation process as compared to publication counts. In such circumstances, the attention of scientists may follow one single (or few) line of investigation, while excluding many research opportunities in a quite ‘autistic’ fashion which one could label a ‘science well’.

6. Tuning the academic reward system for improving the decentralized allocation of attention

In the two preceding sections we have focused on the dynamic properties of the process of scientific knowledge generation and have studied how the structural properties of emerging disciplines vary according to the set of incentives/motivations at play within the community. This led to highlighting that some obviously ‘ill’ dynamics may arise when the academic system of incentives is not well balanced. Nevertheless, we did not try to compare the outcoming disciplines in a systematic manner according to some given efficiency measure. Such a normative study is the purpose of this section. In a first sub-section we introduce the normative criterion and give the first best allocation of efforts. Since it cannot be implemented, the second sub-section introduces a simulation protocol which allows us to analyze how to tune the three types of academic incentives in various situations.

6.1. A social surplus measure and the first best

At each period of the discrete time the discipline (i.e. the tree) generates a social wealth given by

$$W(T^t) = \sum_{i \in V^t} v_i^t, \tag{8}$$

with v_i^t the flow of social wealth provided by area i at period t. Thereby social wealth is assumed to be additive over research areas. We further assume that the social returns to addressing the next problem at hand in a given research area (the first one for new research area) are only function of the number of problems already addressed there, such that $v_i^t = v(\phi_i^t)$. Thus the normative principle implicitly considers that the social returns are independent of both the generality level (d_i) and the audience (s_i^t) of the research area.[16]

In order to specify the shape of this function, we propose the returns of a given problem to be first increasing with the number of problems already addressed while it should decrease once a given

[16] For instance, some arguments would support that general areas would be more wealth providing while some are in favor of more specialized areas. Our ‘agnostic’ modelling option appears most reasonable at least in a first step.

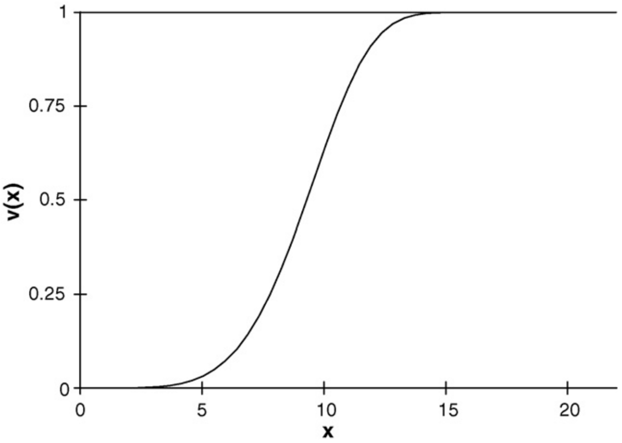

Fig. 7. An example of $v(x)$ as specified in Eq. (9) with $\varphi = 5$ and $\eta = 10$.

level is reached. Even if the first discoveries are often of great importance for further research, they usually do not have a high intrinsic social value. It is only once a reasonable amount of knowledge has been accumulated that marginal improvements begin to generate significant social surplus. But marginal returns are not likely to be always increasing. On the contrary they should decrease once a certain amount of problems has been addressed and tend to zero when the number of problems addressed becomes very large. This expresses the idea according to which there is a limited number of relevant problems to be addressed in given research areas. Formally $v(\phi)$ should be always increasing with $\phi \geq 0$. Its derivative $v'(\phi)$ should also strictly increase when $0 \leq \phi < \eta$ with η some strictly positive parameter ($v''(\phi) > 0$ if $\phi < \eta$). An inflexion point is reached at $\phi = \eta$, and $v''(\phi) < 0$ when $\phi > \eta$. We propose the following Weibull cumulative density function specification for $v(\cdot)$ which respects the above conditions:

$$v(x) = 1 - \mathrm{e}^{-((x/\eta))^{\varphi}}, \tag{9}$$

with parameters φ and η such that $\varphi > 1$ and $\eta > 0$. Parameter φ controls the sharpness of the curve around the inflexion point and η gives the inflexion point.[17] Fig. 7 gives an example of the shape of $v(\cdot)$. According to that specification we have $v(\cdot) : \aleph \to (0, 1)$.[18]

In order to account for the efficiency of the dynamic allocation of attention over research areas we should not limit ourselves to a one-period analysis but rather extend the efficiency measure to a whole large finite or infinite period of time. If we write $\rho > 0$ the society's preference for present, the whole value the discipline generates over its first t periods, actualized at period 0, is given by

$$W_0(T^{\tau} | \tau = 1, \ldots, t) = \sum_{\tau=1,\ldots,t} (1+\rho)^{-\tau} \sum_{i \in V^{\tau}} v(\phi_i^{\tau}). \tag{10}$$

[17] It is a well known property of the Weibull cumulative density function when $\varphi > 1$. It could be easily demonstrated that $v''(x) = 0$ if and only if $x = \eta$, and that $v''(x) > 0$ if $x < \eta$, and $v''(x) > 0$ if $x < \eta$.

[18] This function could thus also be viewed as the probability that at the period considered the accumulated knowledge in the research area leads to a successful use in the economy (say an innovation of normalized unitary value).

We now turn toward determining the first best allocation of scientists' attention over research areas with regard to such a normative criterion. At the first periods, given the initially convex shape of the $v(\cdot)$ function, the social planner will prefer to concentrate all the scientific community's attention on one given area at the time. The returns from addressing the next problem in the already explored area are higher than exploring a first problem in a new area. Since the $v(\cdot)$ function becomes later concave and tends to one, at a given number of problems solved, the social returns from addressing a new problem there will be overcome by having one first problem handled in a new area. It is not that the new problem will *per se* bring a superior amount of social returns, but rather that it will allow society to have the whole series of problems 'behind' being addressed sooner. Once it is reached for the first area, the scientists' attention should then switch to a new area and so on. This number is unique for all areas since so is the $v(\cdot)$ function. It is the optimal number of problems to be treated denoted ϕ^*.

Since this algorithm of sequential allocation of attention will always concentrate all attention on one single area for a fixed period of time, we can write the social value generated as a function of $\hat{\phi}$, the number of problems addressed in each area. It is given by

$$\tilde{W}_0(\hat{\phi}) = \sum_{i=1,\dots,T/\hat{\phi}} \sum_{\phi=1,\dots,\hat{\phi}} (i \times v(\hat{\phi}) + v(\phi))(1+\rho)^{-(\phi+i\hat{\phi})}. \tag{11}$$

T is the social planner's time horizon which can be either finite or infinite. If finite it is assumed to be a multiple of the the number of problems to be treated on each area $\hat{\phi}$. At each period $t = (\phi + i\hat{\phi})$ the flow of social wealth is the discounted sum of wealth from all i research areas on which $\hat{\phi}$ problems have been addressed $(i \times v(\hat{\phi}))$ and of wealth from the currently explored research area for which this level has not yet been reached $(v(\phi), \phi < \hat{\phi})$.

The optimal number of problems ϕ^* is by definition such that: $\phi^* \in \mathbb{N}^*$, $\tilde{W}_0(\phi^*) \geq \tilde{W}_0(\hat{\phi}), \forall \hat{\phi} \in \mathbb{N}$. Such a level is numerically computable.[19] Nevertheless it can obviously not be implemented because it would necessitate that all scientists change their research fields over time which would generate incredibly high moving (learning) costs and certainly a huge burden. Therefore, in the next sub-section, we focus on second best analysis, aiming to tune the typical incentives of the reward system in order to have an 'as efficient as possible' decentralized allocation of attention.

6.2. Fine-tuning the academic reward system

Once the welfare criterion is introduced we look forward to identifying the good combinations of incentives that are tuned through parameters γ, λ and δ on which we again focus. We present the results of two simulation protocols. The first one provides a more intuitive approach while the second one explores the possible combinations of typical incentives more systematically.

In the first protocol, we set α to unity, φ and η as in the numerical example of Fig. 7 and let $\rho = 0.005$. Then, we set $\lambda = 1$ and let γ and δ take different values. For each combination of parameters values, an experiment is performed over 1000 periods. At each period τ, $W(T^\tau)$ is recorded and thus $W_0(T^\tau | \tau = 1, \dots, 1000)$ is computed at the end of the experiment. This is repeated 20 times and we take the mean. We then set $\delta = 1$ and repeat the whole protocol with γ and, λ vary. Lastly, $\gamma = -1$ is set while λ and δ vary. This protocol corresponds to

[19] For $\varphi = 5, \eta = 10$ (numerical example of Fig. 7) and $\rho = 0.005, \phi^* = 12$ whatever the time horizon is finite or infinite. When $T = 1000$, we have $\tilde{W}_0(\phi^*) = 2900$.

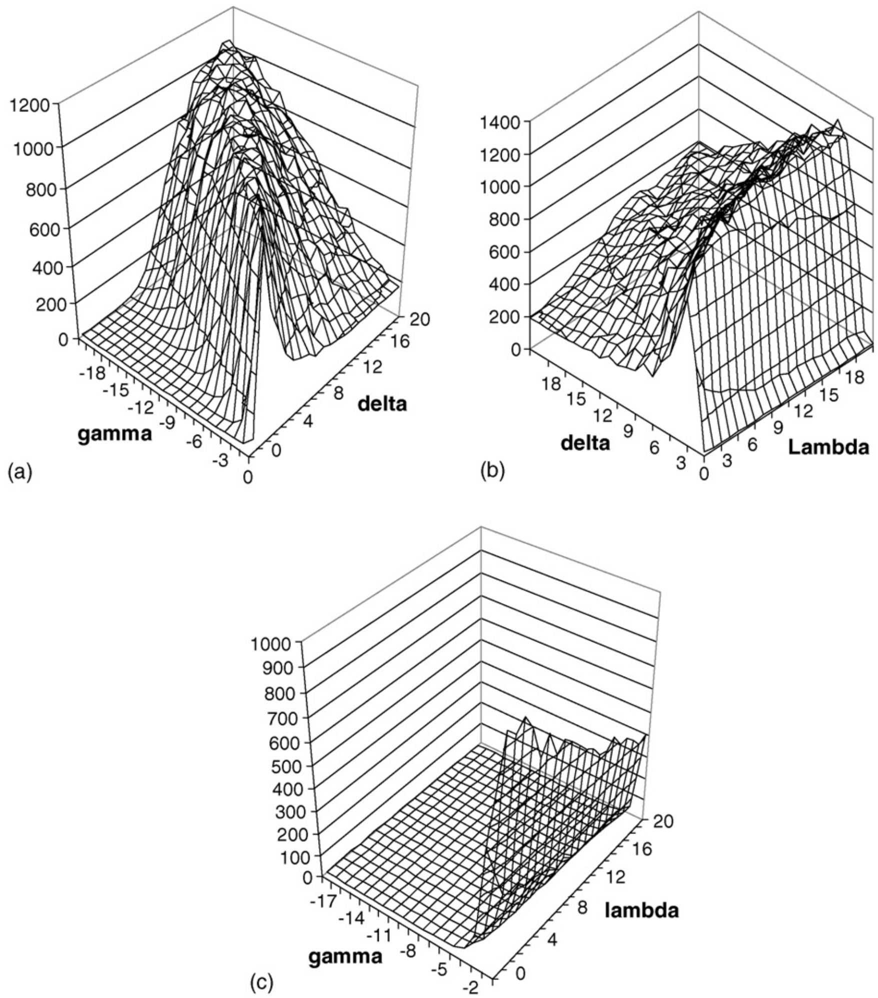

Fig. 8. Mean values obtained for $W_0(T^\tau|\tau = 1, \ldots, 1000)$ over 20 experiments for each combination of the parameters values. In graph a, δ and γ vary and $\lambda = 1$. In graph b, λ and γ vary and $\delta = 1$. In graph c, λ and δ vary and $\gamma = -1$. Other parameters are fixed: $\alpha = 1$, $\varphi = 5$, $\eta = 10$ and $\rho = 0.005$.

$21^2 \times 20 \times 3 = 26{,}460$ simulation experiments. It is designed to give some first evidence on how academic incentives should be balanced. The results are presented in Fig. 8.

We find that when λ is equal to unity, γ should increase linearly and proportionally when δ decreases. That is incentives for pioneering research should linearly balance audience based incentives. When disciplines are very integrated, i.e. when applied areas frequently use and refer to general results, then the rewards for pioneers should be high in order to avoid that scientists' attention over-restrict to 'central' areas and to provide incentives for exploring new areas. Graph b of Fig. 8 shows that, when $\gamma = -1$, there is room for improving welfare by increasing slightly δ and simultaneously increasing sharply λ: strong incentives for specialization could also successfully balance audience based incentives. Since new areas have an audience restricted to themselves (tree 'leaves'), stronger audience based incentives reduce exploration. Therefore, rewarding more

Table 4
Mean (and standard deviation in parentheses) numerical values of the parameters values (γ, λ, δ) and of the relative parameters values $|\gamma + \delta|$, $|\gamma + \lambda|$, and $|\delta - \lambda|$, for subsets of the experiments ranked according to the social surplus generated W_0 (best 1%, 5%, 10%, 20%, 50% and worst half)

	γ	δ	λ	$\|\gamma + \delta\|$	$\|\gamma + \lambda\|$	$\|\delta - \lambda\|$
Best 1%	−15.25 (2.97)	17.75 (2.29)	14.80 (3.37)	3.97 (2.85)	2.66 (1.71)	3.89 (2.88)
Best 5%	−14.35 (3.52)	16.68 (2.81)	14.42 (3.70)	4.47 (3.20)	2.64 (1.94)	3.98 (2.99)
Best 10%	−13.43 (3.88)	15.73 (3.17)	13.88 (4.00)	4.91 (3.40)	2.76 (2.12)	4.15 (3.07)
Best 20%	−12.12 (4.48)	14.40 (3.80)	12.99 (4.47)	5.58 (3.87)	3.08 (2.42)	4.62 (3.38)
Best 50%	−10.12 (5.45)	12.93 (4.59)	10.92 (5.65)	6.92 (4.73)	4.39 (3.49)	5.91 (4.31)
Last 50%	−9.88 (6.60)	7.07 (5.93)	9.08 (6.30)	7.04 (5.17)	9.58 (4.84)	8.05 (5.32)

Other parameters are fixed: $\alpha = 1$, $\varphi = 5$, $\eta = 10$, and $\rho = 0.005$.

specialized areas provides more incentives for exploration since it decreases the relative strength of audience based incentives. The last graph shows that there is no impact of playing on λ in order to balance for γ when $\delta = 1$. Then γ should be set to zero when $\delta = 1$ as graph a of Fig. 8 shows. Nevertheless we are still not able to say if this is a general result or if it is due to the chosen numerical values of δ. The next experiment will show that this latter effect is not robust while it highlights the necessary compensation between the three kinds of incentives.

The previous protocol only explores parameters value spaces when one of them is equal to unity (-1 for γ). We now extend the analysis and look for the best combinations of the three parameters. We are in search of the best regions in the whole box $(\gamma, \lambda, \delta) \in ((-20, 0), (0, 20), (0, 20))$ that we assume to be the parameters value spaces of interest.[20] We run 10 simulations of each discrete combinations of this space ($21^3 \times 10 = 92{,}610$ experiments) and record social surplus scores (W_0) of each run after the usual 1000 periods. We find a mean social value of 782.21, while scores range from a minimum of 2.52 to a maximum score equal to 1843.56. A very simple and robust way to analyze the results is to look at the combinations of parameters which allow us to generate the best scores. Table 4 below gives the mean and standard deviations of parameters values for the best centile, best 5%, best decile, best 20%, best half and worst half of all simulations. We find that the more one restricts to the best scores, the higher the absolute values of the three parameters. The latter increase up to an average of $(\gamma, \lambda, \delta) = (-15.25, 17.75, 14.80)$ in the best centile experiments. This shows that strong traditional incentives of the academic reward system clearly improve the social surplus. Moreover, we compute the absolute value of the difference between parameters (given by $|\gamma + \delta|$, $|\gamma + \lambda|$, $|\delta - \lambda|$) in order to show whether various incentives are either substitutes or complements. It can be observed that the best scores correspond to the lowest differences between parameters amplitude. This supports the idea that the three types of incentives are complements and should thus be provided simultaneously in order to all influence agenda selection while balancing each other.

7. Conclusion

In this paper we have presented an original model of knowledge production within scientific disciplines. It is a graph theoretical model in which knowledge production is sequential. The main question tackled in the paper is how the specific incentives provided by the academic reward

[20] Such parameters values space provide for a sufficiently vast tuning of the relative strength of the relative incentives when one thinks that these parameters enter as power exponents in the reward function given in Eq. (3).

system influence researchers' problem choice and thus shape the stochastic process of knowledge generation within scientific disciplines. Let us sum up the main results obtained.

We first found that the process exhibits a sustained decline in the generation of new areas. We are inclined to compare this evolution and the progressive shift in the growth of scientific knowledge from "little science" to "big science" described by Price de Solla (1963). Our regime change concerns the rarefaction of new areas investigation while Price deals with articles production. It applies to a given discipline while Price's statement concerns the whole body of scientific knowledge (though most empirical confirmations were performed of a single discipline, see. the survey of Fernandez-Cano et al., 2004). Lastly and most important, it is mainly due to incentive reasons rather than to a saturation hypothesis. This phenomenon is caused by the specific reward system in science which leads researchers to seek others' attention. When the discipline grows, the relative rewarding of problems located in already developed fields increases: because their audience becomes larger, contributions to such domains are more likely to be cited. This is not to be seen as *a fortiori* negative: since more knowledge is likely to be produced in larger domains, early contributions there are likely to benefit to many late improvements. That is directly connected to the fact that the citation system traces and rewards knowledge spillovers. Nevertheless, this first result suggests that the rewards for performing pioneering research should be increased when the discipline grows in order to sustain research areas generation.

We next found that the stochastic process exhibits path dependency with regard to the specialization of disciplines. More specialized disciplines tend to become even more specialized through time. We found that this property is enhanced when the concentration of scientists' attention on the most rewarding areas is stronger. In addition to these first series of results, the study of parameters effects allowed us to highlight the possible occurrence of a quite 'autistic' dynamics leading to a 'well' form of discipline having left many research opportunities unexplored.[21] We found that increasing the relative rewarding of pioneering research is again a key leverage parameter because, under such circumstances, it also (unexpectedly) renders general problems more attractive. We argued that such a situation is more likely when the relative rewarding through recording citations is outweighed by publication counts. Thus reinforcing the former mechanism may partly prevent such 'science wells' from occurring.

We also provided a measure for social wealth generation for which we assume additionality over research areas, lower social returns of the very first problems resolution and, a limited number of interesting problems on each area. According to that normative criterion, society would like each area to be in turn explored up to a given level. The first best being not implementable, we studied how the tuning of the typical incentives affects the efficiency of the decentralized allocation of attention. Our results legitimate the academic reward system for orienting scientists' attention. We showed that the three typical incentives of the academic reward system (rewards for novelty, audience and specialization) should be all at play simultaneously and should balance each other. Therefore, if some structural forces that are inherent to the characteristics of knowledge produced or to the historical organization of the community distort the influence of the various incentives of the academic reward system (e.g. prevalence or weakness of one type of incentives) there is room for science policy to correct such distortions.

Nevertheless nobody would obviously consider deriving precise sound science policy measures from the very preliminary model presented above—indeed nor would we. Our goal here

[21] This phenomenon may share some common features with the "disciplinary lock-in" of Windrum and Birchenhall (1998) though, in their model, it is a consequence of the domination of a few research units.

was mainly to show that models can be built to study and simulate the workings of academic communities: models which, once calibrated using empirical evidence (a crucial aspect of the future research agenda in our particular research area), could open the way for more concrete studies. Among the elements that would be worthily considered in further research are the impact of age and career stage on individual choices, the thematic mobility patterns of scientists, the impact of the academic organization in teams, laboratories and universities and, the strategic citation behaviors. For the time being, we can simply say that the studies of imperfect Open Science are just at their beginning.

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