How Do Inventor Networks Affect Urban Invention?*

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Abstract

Social networks are expected to matter for invention in cities, but empirical evidence is still puzzling. In this paper, we provide new results on urban patenting covering more than twenty years of European patents invented by nearly one hundred thousand inventors located in France. Elaborating on the recent economic literatures on peer effects and on games in social networks, we assume that the productivity of an inventor's efforts is positively affected by the efforts of his or her partners and negatively by the number of these partners' connections. In this framework, inventors' equilibrium outcomes are proportional to the square of their network centrality, which encompasses, as special cases, several well-known forms of centrality (Degree, Katz-Bonacich, Page-Rank). Our empirical results show that urban inventors benefit from their collaboration network. Their production increases when they collaborate with more central agents and when they have more collaborations. Our estimations suggest that inventors' productivity grows sublinearly with the efforts of direct partners, and that they incur no negative externality from them having many partners. Overall, we estimate that a one standard deviation increase in local inventors' centrality raises future urban patenting by 13%. We also find that geographically close relations are up to two third more beneficial to inventors than distant ones.

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1 Introduction

It is well known that invention and R&D activities are highly concentrated geographically, even more so than manufacturing employment (Audretsch and Feldman, 1996; Buzard and Carlino, 2013; Carlino et al., 2007). The literature highlights that a critical force for the agglomeration of inventive activities is knowledge spillovers between workers specialized in innovation tasks. Long ago, Marshall (1890) already highlighted that ideas can be shared locally through social and professional interactions. The role of these interactions has since been shown to be crucial in many successful technological clusters (e.g. Saxenian, 1991; Porter, 1998). Jaffe et al. (1993) argue that knowledge flows diminish with geographical distance as citations are more likely to come from the same metropolitan area (MSA) as the cited patents.[1] Other authors make it clear that social and professional connections between inventors who are most often geographically close (Breschi and Lissoni, 2005; Carayol and Roux, 2007) are key determinants of knowledge diffusion (Singh, 2005; Agrawal et al., 2006; Breschi and Lenzi, 2016).

Those findings suggest that social networks between inventors are an important source of disparities in inventive productivity across cities or regions because they facilitate knowledge diffusion. However, to date, the main empirical studies that have exploited the availability of patent data to assess this influence have produced contrasting and somewhat puzzling conclusions. Fleming et al. (2007) and Lobo and Strumsky (2008), using nearly identical US patent data from the late 1970s to 2002, regress, at the MSA level, patent counts against network variables built using co-invention patterns and other controls. Breschi and Lenzi (2016) use EPO patent data of inventors located in US MSAs to build network variables prior to 1999 in order to explain patenting in year 2009. These three studies converge to stress the positive effect of inventor agglomeration. However, they find that the structural characteristics of the co-invention networks[2] have only small effects on urban patenting. Lobo and Strumsky (2008) even find a negative effect of network density on urban invention. Local social proximity, that is, the average of the inverse social distance between a city's inventors, has a small positive effect for Fleming et al. (2007) but no significant effect according to Breschi and Lenzi (2016). Both articles argue that combining local social proximity (for knowledge diffusion) and social cliquishness[3] (for cohesion and cooperation enforcement)

[1] Much other direct or indirect evidence has been provided for the fact that knowledge spillovers arise over small geographical distances (for a recent survey, one may refer to Carlino and Kerr, 2015).

[2] Such networks are built by drawing a link between two agents when they are both listed as inventors of the same patent application.

[3] Often measured by the frequency of closed triangles over the frequency of connected triples. Also called

should positively affect invention, but their results again diverge as the former study concludes negatively while the latter concludes positively.[4] Breschi and Lenzi (2016) find that network proximity between a city's inventors and the inventors located outside the city does not correlate significantly with invention, only that its interaction with local proximity is positive.

These results challenge our conception of how networks affect urban invention. We would have expected that, in urban areas, denser and well architectured webs of connections clearly spur the diffusion of information and ideas between participants, and eventually stimulate their inventive productivity. But does knowledge really (even imperfectly) flow in networks, so that cities which are more connected and which minimize between-inventors distances invent more? A bunch of recent empirical studies suggest that a slightly different story may be true. Azoulay et al. (2010) shows that, following the sudden death of a 'superstar' scientist, his/her direct collaborators face a significant and long-lasting decline in their productivity, and that this effect increases with their intellectual proximity. Using the dismissal of Jewish mathematics professors in Nazi Germany as a source of exogenous variation in university quality, Waldinger (2010) concludes that the mentor's quality affects both the short-term accomplishments and the long-term career achievements of the former PhD students. Borjas and Doran (2015) stress that among the mathematicians remaining in the former Soviet Union after 1990, the only ones who significantly suffered from the loss of their colleagues emigrating to the West were those who lost direct collaborators. These findings highlight the importance of direct and intense collaborations with high-quality partners. They are consistent with the idea that professional networks stimulate knowledge production and invention mainly because, in direct professional collaborations, they emulate early discussions and confrontations of ideas between very active and committed peers, and less because they act as channels for knowledge diffusion.

In this article aiming to empirically analyze how the social networks of inventors affect their performances, we propose microfoundations which are consistent with those basic ideas. We rely on games in which each agent's payoffs essentially depend on his/her action (typically level of effort) and on those of his/her directly connected agents.[5] In this approach, the emulation between connected partners is basically captured by the complementarity between partners' strategies, that is, the productivity of each agent's efforts increases with the efforts of his/her partners. Ballester et al. (2006) first showed that when actions are linear strategic

global clustering in the literature.

[4]Notice that different studies in specific contexts (scientific or artistic productions for instance) are not more conclusive concerning "small-world" effects (e.g., Uzzi and Spiro, 2005; Guimera et al., 2005; Smith, 2006).

[5]There is a broad and long lasting literature in economics on team work, from Marschak and Radner (1972) who focused on communication channels and decision, to others that have considered free riding in groups (for instance Adams, 2006 in a CES production function framework). The network approach has lead to reconsidering this view using graphical games, that is games in which agents interact with their direct neighbors on the graph (see Jackson, 2008 for an overview).

complements (and under some boundary conditions), there exists a unique Nash equilibrium in which agents' actions are equal to their Katz-Bonacich centralities.[6] Technically, our model is more general in that effort complementarity is not necessarily linear.[7] More specifically, our model contains three adjustable basic ingredients: *connectivity*, *synergy* and *rivalry*. *Connectivity* simply presumes that inventor productivity is directly and positively affected by being connected to other inventors. *Synergy* posits that the productivity of an inventor's efforts depends positively on the efforts that his/her partners put into knowledge production. *Rivalry* captures the idea that agents may not benefit a partner's efforts as efficiently when the number of his/her connections increases.[8] In this set-up, equilibrium inventors' outcomes are proportional to the square of a certain form of their network centrality, which, as we will show, is itself parametrized by the degrees of connectivity, synergy and rivalry. This form of centrality is generic, as it nests existing centrality measures such as Degree, Katz-Bonacich and Page-Rank (Katz, 1953; Bonacich, 1972; Brin and Page, 1998).

By bringing this heuristic model to the data, we seek to identify which premises on the way agents affect their neighbors' research productivity, typically which degree of *connectivity*, *synergy* and *rivalry*, best predict future inventions. Our data concern nearly one hundred thousand French inventors and their collaborations for the period 1981-2003, previously cleaned and disambiguated (Carayol et al., 2015), the related information on European patents for the same period, the forward citations made to those patents until 2008, as well as mandatory company survey data from 1985 to 2003. We identify network effects at the level of the local community of inventors by pooling all information at the level of the urban French employment areas (EA) combined with the broad technological field.[9] We estimate a model in which the future patent production of such communities is a function of the average network centrality of their inventors. The structure of the data allows us to include various sets of controls such as EA-technology and time-technology fixed effects as well as several other variables capturing agglomeration economies, which have proven to be important determinants of invention in cities (Fleming et al., 2007; Lobo and Strumsky, 2008; Breschi and

[6] Helsley and Zenou (2014) explore some interesting theoretical implications of this model concerning social interactions in cities. The questions they address are however very different from ours as they compare the periphery and the center and endogeneize the location decision, whereas we focus on the form of complementarity between agents.

[7] Several recent articles have sought to explicitly model the way agents efforts combine in partnerships. Cohen-Cole et al. (2017) have considered the case in which agents interacting in networks exert efforts in different activities. Hsieh et al. (2017) generalize this model allowing linear complementarities between agents' project-specific efforts. Those models use linear production functions.

[8] This idea is reminiscent of the "co-author model" introduced by Jackson and Wolinsky (1996) in which agents divide their time equally in joint bilateral projects undertaken with each of their direct connections.

[9] Recently, regression techniques have been introduced to overcome the estimation issues (such as the reflection problem, Manski, 1993) that arise in individual level estimations (e.g. Bramoullé et al., 2009; Lee et al., 2010; Patacchini et al., 2016; Lindquist et al., 2015). However, in urban and regional economics, scholars often do not work at the individual level because proper identification would at least require the use of rich covariates that are only available at a more aggregated level. We are following this approach.

Lenzi, 2016).

The results show that the inventive productivity of cities is positively and significantly affected by the network structure of its inventors. Our preferred estimation indicates that a one standard deviation increase in local inventors' centrality raises future invention in an urban area and technology field by 13%. Further, no rivalry effect is found but a strong synergy effect is. According to our microfoundations, the results suggest that keeping all other factors constant, a ten percent increase in the efforts of the direct connections of an inventor would raise the social component of his/her productivity of efforts by five percentage points on average.

These results hold across a long series of robustness checks. One of the main concerns we deal with is that, thought we have a rich set of covariates, time-varying unobserved variables might still affect both present network centrality and future invention. However, the effect of inventors' network centrality is robust to the introduction of current performances for predicting a city's future invention, which limits the issue of reverse causality. Further, we pay special attention to the problem of the spatial sorting of inventors which could bias our results if most productive inventors move to places that currently host more central inventors. The results are however not altered by such composition effect as they essentially remain the same when mobile inventors and newcomers are excluded. Other robustness checks deal with the sensitivity of our results regarding star inventors, the Paris region, and the spatial and technological scales of our analysis. Interestingly, the NUTS3 geographical scale allows us to also control for the local R&D spending of companies.

We also provide two extensions to this work. In the first one, we relax the implicit assumption that the network is exogenous. There is now a large literature in economics initiated by Jackson and Wolinsky (1996) which views social networks as intentionally formed. This literature has started to be applied to real data (e.g. Fafchamps et al. (2010)) and in particular the formation of collaborations between inventors (e.g. Carayol et al. (2015), and König (2016)). This creates a potential source of endogeneity: if inventor connections are explained by variables that are correlated with inventors' productivity, we may end up reporting spurious estimates of agents' network characteristics. We propose an estimation strategy to deal with this potential problem. A network is generated between (real) inventors in a preliminary step according to the determinants of link formation that are controlled for in the main equation. Inventors' centrality in this network can thus be used as a consistent instrument for observed network centrality. The results are robust to such form of endogeneity.

The second extension accounts for the role of geographic distance in mediating social relations, as it may alter the strength of the connections between inventors. This enriched setup allows us to obtain interesting new results. We find that geographically close relations are up to two-thirds more beneficial to inventors as compared to connections between distant inventors.

The remainder of the paper is organized as follows. Section 2 describes the construction of our sample and provides some motivating descriptive statistics. Section 3 presents the model which links inventors' productivity to their network. Our empirical strategy is described in Section 4, and the variables in Section 5. The results are presented in Section 6. Robustness checks are reported in Section 7. We deal with endogenous network formation in Section 8 and extend the model to account for the effect of geography on the benefits of social connections in Section 9. The last section concludes.

2 Patent data, sample construction and first motivating empirical insights

In this section, we describe our dataset construction and provide some motivating descriptive statistics about inventor collaborations, inventor concentration and inventor productivity.

Patent data Our starting dataset consists of all European patent applications in which at least one inventor declared an address in France and which were first applied for between January 1981 and December 2003. This represents 125,162 patents. After inventor disambiguation,[10] we know that these patents have been invented by 98,239 distinct individual inventors.

Collaborations as co-inventions Interactions between knowledge workers cannot be directly observed. Instead, as is typically done in network studies using patent data (see e.g., Singh, 2005; Agrawal et al., 2006; Fleming et al., 2007; Carayol and Roux, 2007; Lobo and Strumsky, 2008; Breschi and Lissoni, 2009), the network of social interactions between inventors is drawn from the patent records. Two inventors are considered to be connected if they are jointly listed as inventors of at least one patent. The underlying assumption is that all inventors of a given patent interacted with each other. This assumption is fairly acceptable for most patent teams as the average team size is rather small (2.13 on average). It might however not be realistic for patents involving large teams of inventors as the number of possible bilateral connections evolves quadratically with the team size. For this reason, we withdraw from the sample all patents having strictly more than 8 inventors.[11] Our fi-

[10] Using patent data raises a cleaning issue due to homonymy problems in inventors' names or to spelling errors. Indeed, the proper identification of inventors cannot be neglected since small identity errors are likely to cause significant changes in network measures. For instance, homonymy errors leading to two different persons being considered the same can lead to erroneously linking different communities of inventors. Inventor disambiguation has been performed using the methodology developed by Carayol and Cassi (2009).

[11] This represents less than 0.2% of all patent applications. Note also that our results are robust to changes in this cutoff. Results with different cutoffs are available from the authors upon request.

nal dataset then consists of 124,825 patents, 97,287 unique inventors and 171,587 inventive collaborations over the 1985-2002 period.

Mapping patents to employment areas We locate patents using the inventors' private addresses listed on the patent application.[12] The post-codes of these addresses are then matched to French towns which in turn can be associated with unique Employment Areas (EA). These EAs are statistical constructs based on daily commuting patterns that account for local labor markets.[13] Most take the form of a core town with its surroundings. These EAs cover the whole French metropolitan territory. Note that though some inventors are geographically mobile (about twelve thousand), they mostly remain in the same areas. Nearly 80% of their commutes are of less than 20 km. Finally, when one patent contains inventors living in different EAs, we assign the patent in full count to each of these EAs.

Collaboration in space To study the geographical distribution of collaborations we calculate the geodesic geographical distance separating co-inventors. The first row of Table 1 shows that social connections are highly correlated with geographical space: more than 75% of all the connections are achieved between inventors that live less than 42 km from each other. More than half of the connections are made within the borders of one single EA. Given the weight of Paris and its region (Ile de France), there was a concern that these statistics could be biased. They are thus recalculated when neither, one, or both inventors are located in the Paris region. Interestingly, more than two thirds of the collaborations within the Paris region are also formed within one single EA. This supports the idea that close proximity is important even within larger agglomerations. Moreover, when excluding the Paris region at both ends of the collaboration, we observe that the first three quartiles of the geographic distance are very similar to (or even below) those calculated for the whole set of collaborations. This would suggest that omitting Paris connections does not significantly modify collaboration patterns. However, as the Paris region accounts for a very significant proportion of research and development in France, we should be particularly cautious to ensure that results are not driven by Paris region polarization.

[12] Note that the location of the assignee (the applicant, which is usually the company in the EPO) is not reliable as compared to inventors' addresses because companies with multiple plants or subsidiaries often do not report an address which matches the geographical area where the research was performed (e.g. Jaffe et al., 1993; Thompson, 2006). This bias is particularly strong in France where corporate headquarters are typically concentrated in Paris which, in our database, clusters 60% of the applicants but only 28% of the inventors.

[13] Continental France is split into 297 EAs. More information on these statistical areas can be found in Jayet (1985). In this article, we use the 2010 release of EAs. Note that, since the results could be sensitive to the choice of geographical units, the econometric analysis will be replicated using French NUTS 3 (see Section 7).

Table 1: Geographic distribution of co-inventors over the 1985-2002 period

Distribution of the links	Nb or % of Links	Quartiles (km) 25th	50th	75th	Max Dist. (km)	Share of links within a Single EA
All Collaborations	171,587 (100%)	4.8	14.4	41.5	1099.8	54.0 %
Paris Reg ↔ Paris Reg	54,834 (32 %)	0	8.8	16.9	233.8	65.0 %
Paris Reg ↔ Non-Paris Reg	28,774 (17 %)	29.8	185.4	400.7	729.52	0 %
Non-Paris Reg ↔ Non-Paris Reg	87,979 (51 %)	4.5	13.2	41.1	1099.8	64.6 %

Notes: "Paris Reg ↔ Paris Reg" indicates that both inventors in the pair are located in one of the nineteen EAs that are totally or partly included in the Ile de France Region. "Paris Reg ↔ Non-Paris Reg" means that only one end is located in the Paris Region. None for "Non-Paris Reg ↔ Non-Paris Reg".

Technological fields When filing a patent with the EPO, the patent holder has to assign it to one or several technological classes which correspond to an international patent classification (IPC) code. These IPC codes are associated with one of seven aggregate technological fields following the French Observatoire des Sciences et Techniques (OST) methodology.[14] We find that more than 93% of all the inventors invent in only one technological field. This percentage is reduced to 80% for prolific inventors who produced more than ten patents. Moreover, these shares are the same when only collaborative patents are considered. This suggests that most inventors are specialized in technological fields.

Urban invention The EAs cover the whole French territory, part of which is rural. Since inventive activity is highly concentrated in space (as inventors are) and is a predominantly urban phenomenon, we choose to focus our study on urban environments. Thus, as in (e.g. Fleming et al., 2007; Breschi and Lenzi, 2016) in the US context, we restrict our analysis to the 71 EAs whose core city's population is greater than fifty thousand according to the 1999 census, thereby following the standard definition of EAs given by the US metropolitan statistical areas (Office of Management and Budget, 2010). These urban EAs account for nearly 85% of all patents.[15]

EA-tech In this article, we use the EA-technology field (EA-Tech) as our statistical unit of analysis, that is, the EA combined with the patent's technological field. This allows us to control for technological fields and thus to account for patenting schemes specific to these fields. Further, as we have seen above, these EAs and technological fields tend to correspond to communities of inventors, as collaborations tend to be more organized within EAs and

[14]The seven technological fields are: Electronics, Instruments, Chemicals, Drugs and Medicine, Industrial processes, Machinery and Transport, Consumer goods and Construction. More information on the transition from IPC to OST7 can be found in OST (2010).

[15]Note that the production of patents in non-urban areas is very erratic. More than 50% of non-urban EA-Techs have strictly less than 1 patent per year on average and 90% produce strictly less than 3 patents per year on average. Included them would subsequently yield to very tiny and non-relevant co-invention networks information.

technology fields. Since there are 71 EAs and seven technological fields, we obtain 497 EA-Tech units. Among them, two EA-Techs do not have any patents over the whole time period and are therefore discarded from the sample. Our final sample consists of 495 EA-Techs, whose inventive performances over nearly 20 years we studied. More detailed descriptive statistics on the EA-Tech invention, network and economic characteristics at the EA-tech level will be provided in Section 5.

The distribution of invention The average urban EA-Tech produces 12 patents yearly, involving 14 distinct inventors. As a first sign that invention is highly concentrated, the median EA-Tech produces only three patents involving four inventors. The same observation holds for patent citations. Two illustrative examples of Electronics and Chemicals are provided in Table 2. For both technologies, the top three EAs account for more than half of the patents.

Concentration and productivity Table 2 also shows that the top five cities as regards invention in those two fields significantly differ in terms of their inventive productivity (patents or citation-weighted patents per inventor). Moreover, their productivity differences appear to be unrelated to the number of patents or to the agglomeration of inventors. For example, Roissy has five times less inventors than Paris in Chemicals, but its average inventor is 45% more productive in terms of patents and 47% more productive in terms of citations. In the field of Electronics, the average Saclay inventor is significantly more productive than the average inventor in the other two more concentrated areas. This statement holds when we extend the analysis to all technologies and EAs. Figure 1 shows that a (positive) relationship between average productivity and the agglomeration of inventors cannot be assumed. Agrawal et al. (2014) reach similar conclusions in the field of computers and communications on a cross section of US metropolitan areas.

3 Micro-foundations at the inventor level

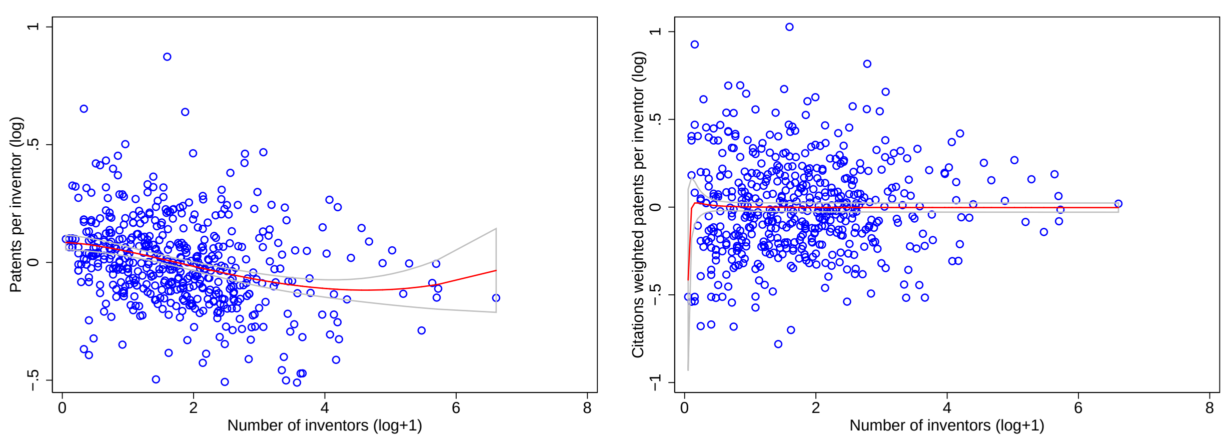
We have seen in the previous section that even though inventors are highly agglomerated in France, their agglomeration clearly fails to explain variations in their performances. We argue that their collaborations, which are often very local, are expected to play an important role. This section intends to provide a simple and flexible view of how inventors' productivity may be affected by their social connections, and hence how the network structure may influence invention. The formal approach is stylized and focused on network-related characteristics. For now, we implicitly assume that the effect the network has on inventors' productivity is independent from any other determinants of productivity, which are discarded from the

Table 2: Top five French cities in Electronics and Chemicals on average over the 1985-2002 period.

EA-Tech	Number of Patents	Shares of Patents	Number of Inventors	Patents per Inventor	Citations per Inventor
Electronics					
Paris	598.8	39.5%	741	0.81	1.75
Grenoble	170.7	11.3%	237.3	0.70	1.49
Saclay	152.9	10.1%	151.2	0.99	2.24
Rennes	50.6	3.3%	61.7	0.75	1.98
Toulouse	42	2.8%	63.8	0.62	1.26
Chemicals					
Paris	246.8	24.9%	305.8	0.81	1.68
Lyon	140.6	14.2%	179.2	0.79	1.57
Saclay	98.7	10.0%	94.5	1.05	2.20
Roissy	68.0	6.9%	57.5	1.18	2.48
Grenoble	38.5	3.9%	58.1	0.67	1.26

Notes: The numbers of patents, citations and inventors are yearly averages. The citations are made by other patents applied in a five-year window after each focal patent. Self-citations are excluded at the inventor and at the applicant levels.

Figure 1: Inventors productivity and inventors agglomeration.



Notes: Each dot represents an EA-Tech. The y and x variables are yearly averages over the 1985-2002 period. The y variable has been previously regressed on technology dummies to expurgate the specific technological features. Similar results are however obtained without such control for the technology. The curves result from the calculation of a fractional polynomial estimation with a 95% interval confidence. The citations are made by other patents applied in a five-year window after each focal patent. Self-citations are excluded at the inventor and at the applicant levels.

analysis. In the empirical sections, we introduce other factors that may affect inventors' productivity and need to be controlled for.

The first subsection describes the model while the second studies the equilibrium and its properties. The last two subsections provide stylized and empirical illustrations.

3.1 The model

Consider a professional network of n agents $i = 1,...n$, typically representing inventors, and whose links between each other are professional connections based on past or present collaborations. The network can be represented by the symmetric square matrix g, whose ith line and jth column entry g_{ij} equals one if inventors i and j are linked, and zero otherwise. The line vector composed of the ith line entries of g is noted g_i. Self-relations are excluded ($g_{ii} = 0$). The number of links of an inventor, also called i's *degree* is $d_i \equiv g_i \cdot \mathbf{1}$, where $\mathbf{1}$ is a n-lines column vector of ones.

Let y_i be the inventive production of agent i. It is modeled as a function of the (research) efforts i exerts, noted e_i. The productivity of efforts is assumed to be constant and divided into an autonomous part and a social part. These two parts are assumed to additively contribute to effort productivity. The autonomous part is normalized to the unity, so that the production of an isolated agent is a one-to-one process which transforms one unit of effort into one unit of outcomes.[16] The social part of effort productivity is noted ψ_i. Agent i's production is thus written as follows:

$$y_i\left(e_i, \psi_i\right) = e_i\left(1 + \psi_i\right). \tag{1}$$

Turning to the utility function, we consider the most simple functional form[17] in which utility equals production minus a quadratic disutility of efforts: $u(e_i, \psi_i) = y_i\left(e_i, \psi_i\right) - \frac{e_i^2}{2}$.

The main ingredients of the model lie in the conceptualization of the social part of the productivity component ψ_i. It is assumed to be (additively) affected by the agents to which agent i is directly connected. We aim to incorporate two effects: synergy and rivalry. Synergy refers to the positive effect that partners' activity has on the productivity of one's effort. Rivalry refers to a local negative effect of partners' involvement in other professional connections. Those two ideas materialize through the following simple expression:

$$\psi_i(g, \mathbf{e}_{-i}) = \lambda \sum_j g_{ij} e_j^{\alpha} d_j^{-\beta}, \tag{2}$$

[16]Setting the default productivity to any other positive value has no implication on the results, as shown in the Appendix.

[17]Again, in the Appendix, we will consider more complex functional forms of the utility function. The inclusion of a parameter tuning effort disutility would imply no significant change in the results.

where $\mathbf{e}_{-i}$ is the vector of all efforts but i's. As g_{ij} is non null only when i and j are linked, only i's partners directly affect his/her productivity. The parameters λ, α and β are assumed to be such that: $\lambda \geq 0$, $\alpha \in [0,1]$ and $\beta \in [0,1]$.[18]

Each parameter influences inventors' productivity and carries a different meaning.

λ **(Connectivity)** scales the linear relation between the network and agents' productivity stemming from the network. If $\lambda = 0$, the inventor's network has no effect on his/her productivity and α and β can no longer be interpreted. The higher λ, the higher the social part of productivity as compared to the autonomous part normalized to one. In a "growth-accounting" view, λ can be seen as a residual. It represents what remains of the social part of effort productivity variation that would not be explained by the variation in partners' efforts and degree (including the effect at the extensive margin due to the variation in the set of partners).

α **(Synergy)** is the elasticity of the social component of i's productivity (ψ_i) to all his/her neighbor j's efforts: $\alpha = \sum_j \frac{\partial \psi_i}{\partial e_j} \cdot \frac{e_j}{\psi_i}$.[19] It gives the social component of effort productivity response, in percentage terms, to one percent increase in all partners' efforts. As such, α thus provides a measure of the degree of synergy between partners, conceptualized as the direct effect of the work intensity of my partners on the efficiency of my own work. If equal to zero, its minimal value, then the efforts of the partners do not matter whatsoever, only the number of partners counts. As long as α is strictly positive, whatever the type of spillover involved, an increase in connected agents' efforts directly enhances effort productivity.[20] When α is maximally equal to the unity, effort productivity increases linearly with partners' efforts (elasticity equal to one). The degree of synergy is assumed not to be greater than unity for consistency requirements, as otherwise equilibrium effort levels become infinite.

β **(Rivalry)** scales the degree of rivalry of direct network benefits. $-\beta$ equals the sum of elasticities of i's productivity to his/her neighbors' degree: $\beta = -\sum_j \frac{\partial \psi_i}{\partial d_j} \cdot \frac{d_j}{\psi_i}$. A one percent decrease in all i's degree of connections increases the social component of i's productivity by β percent. An elasticity equal to -1 ($\beta = 1$) corresponds to the maximal possible rivalry. In that case, the sum of direct network externalities stemming from any connected agent j is fixed, to be divided equally among all his or her

[18]From a technical point of view, this setup generalizes the model proposed by Ballester et al. (2006), which is the special case of the linear effect of neighbors efforts on productivity (when $\alpha = 1$) and no rivalry ($\beta = 0$).

[19]It can be calculated as follows: $\sum_j \frac{\partial \psi_i}{\partial e_j} \cdot \frac{e_j}{\psi_i} = \sum_j \frac{g_{ij}\lambda\alpha e_j^{\alpha-1} d_j^{-\beta} e_j}{\sum_j g_{ij}\lambda e_j^{\alpha} d_j^{-\beta}} = \frac{\sum_j g_{ij}\lambda\alpha e_j^{\alpha-1} d_j^{-\beta} e_j}{\sum_j g_{ij}\lambda e_j^{\alpha} d_j^{-\beta}} = \alpha$, if $\lambda > 0$. If λ is null, the elasticity cannot be calculated.

[20]If α were negative, then efforts would be strategic substitutes. This is not relevant in our application, but could fit other situations.

partners.[21] In that case, if an agent experiences an increase in his or her neighborhood size, its accumulated direct impact on his/her neighbors' productivity remains constant. Conversely, if β becomes null, then productivity is totally inelastic to the partners' degree. Each inventor contributes fully to each of his/her collaborators. There is no rivalry effect in this case, as agents do not suffer from their direct connections having more acquaintances, all other factors kept constant.

It should be noted that the effects we have examined so far are only direct ones, which are likely to 'spread' along the inventor's direct connections. For instance, an increase in an inventor's efforts will raise the productivity of his/her direct connections, which will lead them to increase their efforts and this in turn will raise the productivity of their neighbors (including himself), etc. The efforts that result from all such interactions in the network are obtained at the equilibrium, studied in the next subsection.

3.2 Equilibrium

We now look at the equilibrium efforts and subsequent inventors' productivity. If each inventor maximizes his/her utility while taking the efforts of all other inventors as given, then the resulting interior Nash equilibrium is characterized by the following n best responses:

$$e_i = 1 + \lambda \sum_j g_{ij} e_j^{\alpha} d_j^{-\beta}, \forall i. \tag{3}$$

The equilibrium research efforts can thus be written as a function of the network and of the parameters: $e_i \equiv c_i(g, \lambda, \alpha, \beta)$. The system of equations given in (3) generalizes well-known forms of centrality, such as the degree centrality (Bavelas, 1948), the Katz-Bonacich centrality (Bonacich, 1987) or the Page-Rank centrality (Katz, 1953; Brin and Page, 1998). These existing centrality measures can be obtained for different admitted values of parameters α and β. Table 3 presents some nested typical forms of centrality. As agents' equilibrium efforts are equal to their centralities, typical assumptions about the social component of the productivity function lead to different effort levels. For instance, if no synergy or rivalry is at play, agents' efforts would be equal to their degree centrality. At the other end of the spectrum, agents' efforts would be equal to their Page-Rank centralities if both synergy and rivalry are maximally unitary.

Combining Equation (1) and Equation (3), it turns out that equilibrium production is given by $y_i^* = (e_i^*)^2 = \left(1 + \psi_i\left(g, \mathbf{e}_{-i}^*\right)\right)^2$. Substituting equilibrium efforts for centralities thus gives:

$$y_i^* = c_i^2(g, \lambda, \alpha, \beta). \tag{4}$$

[21] As in the co-author model introduced by Jackson and Wolinsky 1996, without considering what they call the "synergy effect".

Table 3: Existing centrality measures nested in the centrality defined by Equation (3). The last column provides the formula of the centrality when the parameters α and β are set as in the first column.

(α,β)	Centrality name	Definition
$(0,0)$	Degree centrality	$c_i = 1 + \lambda d_i,\ \forall i$
$(1,0)$	Katz-Bonacich centrality	$c_i = 1 + \lambda \sum_j g_{ij} c_j,\ \forall i$
$(1,1)$	Page-Rank centrality	$c_i = 1 + \lambda \sum_j g_{ij} \frac{c_j}{d_j},\ \forall i$

Therefore, i's equilibrium production is equal to the square of its network centrality, which depends on the network g and on the three parameters λ, α and β.

Existence and unicity Is the centrality measure defined by Equation (3) well defined, in the sense that it has a positive solution so that the equilibrium efforts and productivities can be computed? Moreover, is it unique? The answers to these questions depend on α. When it is equal to one, the system of equations in (3) is linear and has a particular behavior as compared to the sublinear case. In short, an equilibrium exists and is unique if λ is sufficiently small (Ballester et al. (2006)).[22] In contrast, as soon as α is strictly less than unity, there is no longer a restriction on λ to obtain existence. Moreover, as Theorem 1 states, the unicity of the solution is guaranteed.

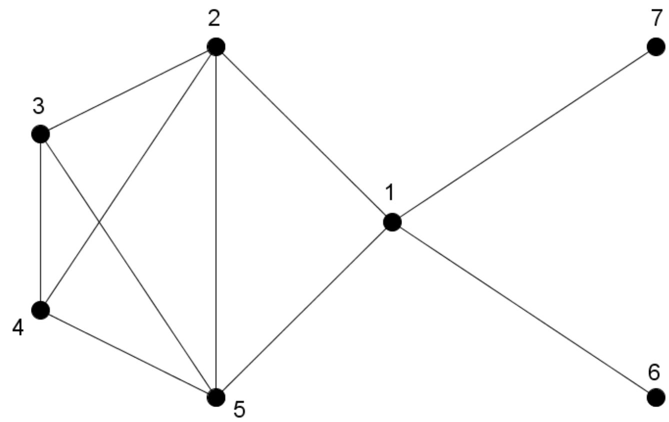
Theorem 1. When $\alpha \in [0;1[$, for any g, β and for any positive λ, the system defined by Equation (3) has a unique positive solution.

The complete proof of Theorem 1 is given in the Appendix. This theorem will be very useful in the empirics, as the sublinear case is consistent with the data. In practice, the unique solution can be computed via an increasing sequence. Let c_i^k denote the centrality of inventor i at iteration k (the parameters α, β and λ are omitted for readability). Each centrality is first initialized to 1 and then the following calculus is performed until numerical convergence:[23]

[22] See the Appendix for more details.

[23] In this paper, whenever this network centrality is to be computed, we stop the algorithm when the maximum absolute difference between two successive centralities ($\max_i\{c_i^{k+1} - c_i^k\}$) is smaller than 0.0001.

Figure 2: Stylized example of an inventor network.



Network.PNG

$$c_i^{k+1} = 1 + \lambda \sum_j g_{ij} \left(c_j^k\right)^\alpha d_j^{-\beta}. \tag{5}$$

At the end of the process, each inventor's centrality respects the definition of Equation (3) up to a negligible numerical error.

3.3 Stylized example

We first use a stylized example to illustrate how typical individual network-positions rank for different traditional centralities nested in our definition. This should clarify that different premises on how agents are influenced by their connections lead to distinct predictions in terms of productivity at equilibrium. We compare several typical centralities computed for the network represented in Figure 2. This figure depicts a network consisting of seven agents, including four agents fully connected with each other (agents 2, 3, 4 and 5) and two having only one connection (nodes 6 and 7). The last agent, 1, is connected to these two groups. We focus on four "typical" agents: 1, 2, 3 and 6. Table 4 reports degree, Katz-Bonacich and Page-Rank centrality. The parameter λ is here set to 0.25 but many other strictly positive values for this parameter could have been chosen for it to respect the existence and unicity conditions. As it only marginally affects the way agents' centralities rank, we do not consider other values and focus on synergy and rivalry effects captured by α and β.

If $(\alpha,\beta) = (0,0)$, so that there is no synergy or rivalry at play in the network, the productivity gain from any connection is the same, equal to λ. The centrality of the agents then relies only on their number of connections. In this case, agents 1 and 2 have four connections and therefore have the highest centrality, followed by agents 3 and 6. In the case of linear synergy but no rivalry, $(\alpha,\beta) = (1,0)$, one's productivity depends positively on the partner's effort, and as the rise in productivity spurs one's effort, this new effort will in turn increase

Table 4: Centrality measures, as defined by Equation (3), for the nodes depicted in Figure 2.

		Centrality	
	Degree	Katz-Bonacich	Page-Rank
Agent	$(\alpha = 0, \beta = 0)$	$(\alpha = 1, \beta = 0)$	$(\alpha = 1, \beta = 1)$
1	**2**	6.08	**1.73**
2	**2**	**7.65**	1.41
3	1.75	6.43	1.28
6	1.25	2.52	1.10

Notes: The value of λ is here set to 0.25.

the partner's productivity, which will increase the partner's effort, etc. Therefore, linear synergy implies that agents who are strongly interconnected in cliques benefit the most from this type of effects. Despite having the same number of connections as agent 1, agent 2 has the highest centrality because he/she is highly connected to highly connected neighbors. Moreover, agent 3, whose degree is lower than agent 1, becomes more central than agent 1, as he/she strongly benefits from being involved in a fully connected clique. As compared to the previous case, when synergy is linear and rivalry is maximal, that is, when $(\alpha, \beta) = (1, 1)$, connections to more isolated agents become relatively more effective. And hence in that case agent 1 surpasses agents 2 and 3 so that agent 1 exhibits the highest centrality.

3.4 Empirical example

Let us consider three distinct local communities of inventors defined as the set of inventors in the 'chemicals' field for the period 1991-1995, each located in a different city area (three distinct employment areas). For each community, we compute the average squared centralities of their inventors. The centralities are of course computed in the whole collaboration network, that is, when all the links between all French inventors are taken into account (any city and any technology). The average centralities for the three cities are given in Table 5. Interestingly, the three communities rank differently depending on the type of centrality. The Orly community has the highest average degree centrality. The highest average Katz-Bonacich centrality is found for Nantes whereas Saint-Étienne inventors have the highest average Page-Rank centrality. We see here that distinct assumptions concerning the manner in which agents benefit from their connections may lead to different predictions on the relative performance of local communities. If one believes that neither synergy nor rivalry is at play, the average inventor in the Orly community should be more productive. If synergy is assumed to come into play linearly, an additional full rivalry assumption would lead to a prediction that inventors in the Saint-Étienne chemicals community are more efficient. And

Table 5: Average squared centralities for several urban (EAs) networks of inventors in the technological field of ‘Chemicals‘.

		Centrality		
Community Name	Number of Inventors	Degree ($\alpha = 0, \beta = 0$)	Katz-Bonacich ($\alpha = 1, \beta = 0$)	Page-Rank ($\alpha = 1, \beta = 1$)
Orly	53	**1.53**	1.91	1.08
Nantes	33	1.45	**3.44**	1.07
Saint-Étienne	43	1.36	1.52	**1.11**

Notes: The value of λ is here set to 0.04 as it is close to (but still less than) the inverse of the largest eigenvalue of the global network (the three communities belong to the same network).

so on and so forth. In fact, all possible assumptions can be screened and their associated predictions can be confronted with the observed future productivity of these communities. We are seeking the premises, fully contained in specific values of parameters λ, α and β, which best explain the observed productivities of these communities. The next section provides an empirical methodology for this purpose.

4 Empirical strategy

4.1 An aggregated production function approach

The production of inventions in each EA-Tech is assumed to follow similar patterns: as in standard regional knowledge production function approaches, the innovative outcomes are obtained from a common production function with similar elasticities across units of observation (Fleming et al., 2007; Lobo and Strumsky, 2008). The main input considered here is the contribution of inventors, which is assumed to constitute the backbone of invention production. The basic relation describing urban invention production is thus given by the following equation:

$$Y_{a,f} = A_{a,f} \cdot l_{a,f}^{\tau}, \tag{6}$$

in which $Y_{a,f}$ is the innovative output of the urban EA a and technological field f, $A_{a,f}$ contains the specific factors affecting urban inventive production and $l_{a,f}$ is the sum of the contributions of all inventors associated with the EA-Tech; τ is a positive parameter.

If we let $Inv_{a,f}$ denote the set of inventors associated with an EA-Tech, we are able to introduce the individual inventive contributions of local inventors as defined in the previous section, as follows:

$$l_{a,f} = \sum_{i \in Inv_{a,f}} e_i \left(1 + \psi_i(g, \mathbf{e}_{-i})\right). \tag{7}$$

The efforts agents exert and the direct influence of their network neighbors are however typically not observable in the data. Our micro-foundations suggest that an agent's effort is affected by his or her network so that, in equilibrium, the network-related production is equal to the square of the agent's network centrality (cf. Equation (4)). This centrality depends on the inventors' network and on the three parameters of connectivity, synergy and rivalry. In equilibrium, we thus have:

$$l_{a,f} = \sum_{i \in Inv_{a,f}} c_i^2 \left(g, \lambda, \alpha, \beta\right). \tag{8}$$

In this equation, it is clear that variable $l_{a,f}$ is in fact the combination of two elements: 1) a size effect, since it is the sum of all inventors associated with the EA-Tech, and 2) a network effect, since the production of each inventor is assumed to depend on their network position. However, as Bettencourt et al. (2007) has shown, the number of inventors is a major determinant of patent production and is tightly linked to city size. In order to identify the network effect, it should be separated from the size effect. In consequence, in Equation (6), the variable $l_{a,f}$ is broken down as the product of the number of inventors in the EA-Tech, $\#Inv_{a,f}$, and their average equilibrium contribution, noted $\overline{c_{a,f}^2 \left(g, \lambda, \alpha, \beta\right)}$:

$$Y_{a,f} = A_{a,f} \cdot \#Inv_{a,f}^{\tau} \cdot \overline{c_{a,f}^2 \left(g, \lambda, \alpha, \beta\right)}^{\tau}. \tag{9}$$

4.2 Estimation procedure

As explained in the next section, urban invention production will be measured in terms of patents or patent-citations, which are count variables. A natural way to estimate the model is to use a Poisson model. Indeed, contrary to linear models which lead to biased coefficient estimates when dealing with count data, a Poisson model copes suitably with this issue and also allows us to deal with over-dispersion (see e.g., Santos Silva and Tenreyro, 2006). Further, to limit the problem of omitted variables and to fully exploit the panel structure of our dataset, we employ a fixed-effect Poisson estimation where the unit of observation is the EA-Tech, so that every time-invariant unobservable effect specific to the employment area and the technological field will be controlled for. Further, Time-Tech dummies are also introduced to control for exogenous shocks that could affect the production of inventions in a specific technology. To avoid simultaneity issues, the dependent variable is forward-lagged in $t+1$ so that the explanatory variables explain the production of the subsequent year (as in Fleming et al., 2007).

Based on this large set of fixed effects, the identification of the effect of inventors' centrality on urban invention then hinges on within-EA-Tech variations, net of any technology-time effect. Therefore, we wonder whether the variation in the centrality of inventors in a given

EA-Tech (e.g. Paris-Chemistry) has any influence on the future invention production of this EA-Tech.

Formally, the equation that will be estimated is:

$$E\left(Y_{a,f,t+1}\right) = d_{a,f} \cdot d_{f,t} \cdot \prod_{k} X_{k,a,f,t}^{\theta_k} \cdot \#Inv_{a,f,t}^{\tau} \cdot \overline{c_{a,f}^2\left(g_t, \lambda, \alpha, \beta\right)}^{\tau} \tag{10}$$

where $Y_{a,f,t+1}$ is the dependent variable, and $d_{a,f}$ and $d_{f,t}$ are EA-Tech and Time-Tech dummies for the EA a, the technological field f and the year t. The k-indexed variables in $X_{k,a,f,t}$ are all other determinants of urban patent production which include agglomeration economies variables as well as other network-related controls. According to our empirical model, we set the elasticity of the average squared centrality, τ, to be equal to the elasticity of the number of inventors.[24]

The parameters of interest in our approach (α, β, λ) cannot be estimated by traditional linear techniques. Indeed, changing the value of any of these parameters implies non-trivial changes to every inventor's network centrality. Stated differently, the network centrality variable cannot be expressed as a linear combination of its parameters with some other exogenous variables. To cope with this issue, we apply nonlinear estimation techniques. Similarly to models with linear right-hand sides, the estimated coefficients are simply the set of parameters that maximizes the likelihood as follows:

$$\arg \max_{\boldsymbol{d}, \boldsymbol{\theta}, \lambda, \alpha, \beta, \tau} \mathcal{L}\left(\mathbf{Y} | \boldsymbol{d}, \mathbf{X}, \boldsymbol{\theta}, Inv, \lambda, \alpha, \beta, \tau\right).$$

The interpretation of the parameters along the model defined in Section 3 is valid only for positive values of these parameters. Further, we restrict α as strictly lower than 1, because if α were equal to 1 then the centrality measure would not be defined for any positive λ (see Section 3.2), which would impede the estimation procedure. Consequently, the estimation runs with the following constraints: $\lambda \geq 0$, $\alpha \in [0; 0.99]$, $\beta \geq 0$.

Finally, even though these parameters enter the model in a nonlinear form, they end up being asymptotically normally distributed (see Wooldridge, 2010, theorem 12.3, p. 407). In every estimation, we will report standard-errors clustered by EA-Tech. As in standard maximum likelihood models, the estimates are obtained by using a maximization algorithm. In this context, the variable $\overline{c_{a,f}^2\left(g_t, \lambda, \alpha, \beta\right)}$ needs to be computed anew at each iteration of the maximization process. To make this estimation, we used the statistical software R in combination with the package FENmlm which estimates maximum likelihood models with

[24]Note that setting the elasticity of the average squared centrality to other values implies no change in our results. Indeed, we replicated the main analysis with elasticities ranging from 0.2 to 1.0 and this did not alter our main findings regarding *i*) the centrality coefficients and *ii*) the average impact of inventors' centralities on future EA-Tech invention. The results are reported in the Appendix.

Table 6: Illustration of the estimation procedure: Four sets of outcomes and the estimated parameters of Equation (11) for each set of outcomes.

	Node							Estimated Parameters		
	1	2	3	4	5	6	7	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
Outcome 1	5	3.5	3	2	3	1	0	1.5	0	0.34
Outcome 2	2.5	5	5	3	6	1	0	0.28	0.985	0
Outcome 3	5	7	3	4	8	1	2	0.42	0.42	0
Outcome 4	5	4	2	1	3	6	7	0	0.28	0.39

fixed effects and which admits nonlinear right hand sides. A simple illustration of this parameter estimation is provided in the next subsection.

4.3 How does the estimation of parameters work? An example

This section is intended to provide an example of how we estimate the centrality parameters λ, α and β. Here we introduce a simple example to illustrate how this non-linear estimation works.[25]

First, consider, as in Section 3.3, the simple network depicted by Figure 2. Then assume that each node produces a specific outcome and that we aim to estimate the link between the node's centrality and its outcome. The estimated relationship is then the following:

$$E\left(outcome_i\right) = constant \times Centrality_i^2\left(\lambda, \alpha, \beta\right), \tag{11}$$

where subset i refers to the node.

The estimation of this equation using a Poisson model would provide centrality parameter estimates such that the nodes with the highest centrality would also be the ones with the highest outcome.

To illustrate this point we create four different sets of outcomes, each providing a different ordering of the nodes. For each of these sets of outcomes, depicted in Table 6, we then estimate Equation (11) via Poisson maximum likelihood and we also report the estimated parameters in Table 6. The main point is that each set of outcomes leads to a different set of estimated centrality parameters. How the parameters are estimated is best described by Figure 3 which depicts the outcome of each node (y-axis) as well as their centrality for the estimated parameters (x-axis).

This figure confirms that the method has estimated centrality parameters so as to provide the highest centrality to the nodes with the highest outcome.

[25] The full R-code to run this example is given in Appendix 10.

Figure 3: Illustration of the estimation procedure: it shows how the estimated parameters depend on the outcomes.

Notes: This graph represents the results of the regression described by Equation (11) via Poisson maximum likelihood. All regressions are based on the same network depicted by Figure 2.

Note that for the fourth set of outcomes, where nodes 6 and 7 have the highest outcome, the estimated value of λ is 0 since the centrality parameters were not able to rank these two nodes high. This situation means that the centrality is irrelevant in explaining this outcome distribution and therefore, as $\hat{\lambda}$ is equal to 0, the two parameters $\hat{\alpha}$ and $\hat{\beta}$ cannot be interpreted.

The results in Section 6 relate to the same idea: the estimated parameters are such that the EA-Techs with the highest future inventive output have the highest centrality of inventors. Similarly to this example, if the centrality measure were not relevant, then λ would have an estimated value of 0.

5 Variables

5.1 Dependent variables

The measure of a city's inventive output will be drawn from patent counts. However, the number of patents alone may not be sufficient as patents vary greatly in quality (see e.g., Trajtenberg, 1990; Lanjouw et al., 1998; Hall et al., 2005). A way to account for patent quality is to measure how much the knowledge embodied in a patent has been used in later patented innovations. When a patent is applied for, it has to reference prior art (see e.g., Criscuolo and Verspagen, 2008, for a review). A positive relationship between patent value and citations received has been demonstrated in various studies (see e.g., Trajtenberg, 1990; Harhoff et al., 1999; Hall et al., 2005). Thus, in order to have a finer grained measure of invention, each patent will be weighted by one plus the number of citations it receives, in line with various studies dealing with urban invention (e.g., Agrawal et al., 2014; Kaiser et al., 2015). In a sense, the patent is itself considered as its first citation, a procedure which has the advantage of not giving a zero mass to a non-cited-patent. This dependent variable is called *number of citation-weighted patents*, noted CW. However, to ensure that the results do not rest upon our choice of dependent variable, we also run the econometric analysis on the *number of patents* (P) and the *number of citations* (C). The former variable reflects the idea of quantity of urban inventive production, while the latter captures the quality of patents. In fact, the main dependent variable combines the two dimensions ($CW = C + P$).

A 5-year window is used to construct the number of citations a patent receives, allowing this number to be comparable across patents from different years. As the most recent patents from our sample are from 2003, we need information on citing-patents until 2008. Further, as the aim is to depict the quality of a patent, the citing-patents should not be restricted to French patents only.[26] The number of citations a patent receives is defined as the number of EPO-patents whose application date lies in the five following years and that cite the application number of the cited patent.[27] Further, in order to avoid citations due to factors unrelated to quality, we withdraw every citation coming from patents either from the same inventor or from the same company.[28] Patents are assigned in full count to each EA in which at least one of the inventors reside. Explicitly, the location of the patents is based on the inventors' addresses, so that the dependent variable for EA a and technological field f will be the number of citation-weighted patents filed in year $t+1$ in technological field f that have at least one inventor located in area a.

5.2 Network-based variables

We start with two comments that apply to all network-based variables. First, these variables will be constructed using a five-year window: from $t-4$ to t. This period of time is used to gather enough information on the network patterns of the EA-Techs as patenting can be considered as a rare event (Lobo and Strumsky, 2008). Second, when, for some EA-Tech, no patent has been produced in the five-year-window, so that some network-based variables cannot be computed (e.g. the *average team size*), we set these variables to their minimal value, and to 0.01 if the minimum is 0 (see Fleming et al., 2007).

Centrality Our main explanatory variable is, as shown in Section 4.1, the average squared network-centrality of the inventors of a given city-technology. This variable is built in two steps.

In the first step, the network-centrality of all inventors is computed by using the whole co-invention network in a 5-year window. The network consists in all collaborations between inventors having patented between years $t-4$ and t, no matter the technological class or the EA. This network is built by assigning a link between each pair of inventors having co-patented at least once during that period. Then, from this network and for possible given values of parameters α, β and λ, we compute the centrality of each inventor $c_i(g_t, \alpha, \beta, \lambda)$,

[26]The citations-related data are drawn from the CRIOS-Patstat database which compiles data on all EPO-filed patents (see Coffano and Tarasconi, 2014, for a description of the database).

[27]The 5-year window is accurate to the day. As the day, month and year of application are available for each patent, we are able to keep only the citing-patents which were filed no later than 1,825 (365×5) days after the cited-patents.

[28]Thanks to the algorithm from Pezzoni et al. (2014), each patent in the CRIOS-Patstat database has an identification number for the inventors who filed it and the companies which own it. The 'self-citations' were cleaned thanks to those identification numbers.

according to the system of Equations in (3).

The second step consists in the aggregation of these inventors' centralities. Each inventor is assumed to contribute *fully* to each EA-Tech he/she has patented in. If a person has moved or has invented patents in different technologies in the considered period, it will be counted in each corresponding EA tech. The average squared centrality among all inventors having patented at least once in the EA-Tech considered over the 5-year window period:

$$\overline{c^2_{a,f,t}\left(g_t,\alpha,\beta,\lambda\right)} = \frac{1}{\#Inv_{a,f,t}} \times \sum_{i\in Inv_{a,f,t}} c_i^2\left(g_t,\alpha,\beta,\lambda\right),$$

where $Inv_{a,f,t}$ (resp. $\#Inv_{a,f,t}$) is the set (resp. number) of inventors having patented in EA a, technological field f and years $[t-4,t]$. Note that for EA-Techs with no inventor in a given window, this centrality is not defined. We thus assign to it the value 1, as it is the minimal value possibly attained by the centrality.

Network covariates The main input of patent generation is creative individuals, and in this subset of the population, the most committed individuals in invention activities, the inventors themselves (Fleming et al., 2007). The variable *inventors* is the number of inventors having patented at least one patent in the EA-Tech over the 5-year period ($Inv_{a,f,t}$). This variable also aims at capturing the effects of the agglomeration of innovative activities.

Patent teams are not locally bounded: they can be the outcome of collaborations between inventors located in different urban areas. If so, the number of inventors of an EA-Tech as a control may be not sufficient to capture the inputs to knowledge creation as it would neglect the inventors outside the EA-Tech who have also contributed to producing the patents in the area. To control for this, we include the variable *share of outside collaborators* which is the number of external (to the area) inventors divided by the total number of inventors having participated in the patents in the area.

The distribution of the patent resources among different technologies may influence the efficiency of knowledge production. If agglomeration economies are at work, the concentration of patents in some particular technological fields may enhance the productivity of research in those fields. Thus we include the variable *technology Herfindahl*, defined as the Herfindahl index of the patents produced in the area distributed among 30 technological classes denoted c.[29] This variable is defined at the EA-year level and its formal definition is $\sum_{c=1}^{30} s_c^2$, where s_c is the share of patents in the technological-class c.

The econometric analysis will control for the specificity of the technological fields with EA-Tech dummies. Yet, even when controlling for a technology, some EA-Tech may still be

[29] The classification leading to 30 technological classes, referred to as OST30, is based on the IPC code of the patents and is a finer grained version of the OST7 classification. See OST (2010) for more information.

specialized in specific fields within a given technology which are more recent and more fertile in new ideas and patents. Those technologies are possibly less likely to cite old knowledge. To control for this effect, we include, as in Lobo and Strumsky (2008), the *technology age* variable which is the average number of references cited by the patents produced in the EA-Tech.

Another important issue stems from the very nature of the collaboration network data we are using. These data are a bipartite graph in which the connections between inventors and patents are observed. The connections between inventors are not directly observable but reconstructed. Two inventors are assumed to be connected whenever they co-invent a patent. This is quite an acceptable assumption to make as inventor teams are usually small (the median is equal to two). However, in this context, team size has a large influence on the network structure since each inventor within a team is connected with all the other inventors of this team. The increase in team size may still raise inventors' average centrality by increasing their number of connections. In consequence, if larger teams produce more patents, and if larger teams also imply higher centrality, then the effect captured by the centrality variable may be spurious. To control for such team size effects, we introduce in our regressions the variable *average team size*, defined as the average number of inventors per patent produced in the EA-Tech-year.

5.3 Agglomeration variables

Finally, we also integrate economic variables in order to account for industry-related agglomeration economies. To do so, we use plant-level data stemming from French annual business surveys over the period 1985–2003.[30] These mandatory surveys provide information regarding employment for all manufacturing firms of more than 20 employees. The precise location along with the industrial sector and the level of employment of each French plant of these firms are also reported. We create, for each EA, the variables *number of industry workers* and *number of plants of more than 200 employees*. The former variable aims to control for the employment density in the industrial sector of the area which has proven to affect inventive production in cities (Carlino et al., 2007). The second one seeks to capture the effects of a potentially higher level of effort in R&D undertaken by large firms (Lobo and Strumsky, 2008). Last, we introduce an index of *employment diversity* in the EA to account for the potential effects of local industrial diversification at the city level, as in Carlino et al. (2007). This index of *employment diversity* is based on a Herfindahl index at the 3-digit sectoral level. It is defined by $\ln(1/\sum_s s_{a,s}^2)$ where $s_{a,s}$ is the share of workers in sector s in city a.

[30]The sources are the data from the 'Enquetes Annuelles d'Entreprises', which are collected by the French Ministries of Industry, Agriculture and Food, jointly with the French national statisitcs institute (INSEE).

5.4 Descriptive statistics

Our sample consists of 495 EA-Techs and 18 years (1985–2002 for the explanatory variables and 1986–2003 for the dependent variables). Table 7a presents the summary statistics for the main variables. The correlation between the variables are reported in Table 7b. The highest correlations are of 90% between the number of workers and the number of large plants, and of 92% between the average team size and the technology age. As these variables are used as controls, we keep them in the sample.

6 Main Results

The results of the Poisson estimation are reported in Table 8. We focus first on Model (1) which is a benchmark model excluding network centrality variables, before going on to comment on the main results in Models (2) to (4). As usual in studies on urban patenting, the number of inventors has a strong positive effect. We find that a 10% increase in the number of inventors leads to a 3.5% increase in future urban patenting. The negative effect of the average patenting team size is in line with the literature (e.g. Lobo and Strumsky, 2008). The estimates suggest that a 10% increase in the average team size in the EA-Tech would imply a decrease of 2.6% in urban patenting. Having access to knowledge from outside the city should be valuable since it creates new possibilities of knowledge combination. Accordingly, the share of inventors from other cities taking part in the EA-Tech patents has a positive and significant coefficient. This result is also in line with the previous studies of Fleming et al. (2007) and Lobo and Strumsky (2008). Specialization at the city level, as measured by the Herfindahl technology, has a positive and significant effect on urban patenting. The age of the technology developed in the EA-Tech, approximated by the average number of references to older patents, has no significant effect. Regarding the agglomeration variables, we observe that the number of large plants and the diversity index of the workforce are not significant whereas the number of workers within the EA increases urban patenting.

We now turn to Model (2) which contains the baseline results of the paper. Does the structure of the inventor network influence future urban invention? The positive and significant coefficient of connectivity λ, indicates that inventors benefit from being connected in the network. It provides a first and reassuring result as it supports the idea that the centrality of inventors sustains innovation. Indeed, it would be misleading to consider that its low estimated value ($\hat{\lambda} = 0.07$) implies that inventor networks play a marginal role. In fact, the global magnitude of the network effect, captured as the effect of the squared centrality on urban invention, can be computed considering within-sample variation in inventors' squared centrality, as estimated in the model ($\overline{c^2_{a,f}\left(g_t, \hat{\alpha}, \hat{\beta}, \hat{\lambda}\right)}$). We look at the increase in future urban invention when the inventors in an EA-Tech increase their centrality by one standard

Table 7: Descriptive statistics and correlations at the EA-Tech level.

(a) Descriptive statistics.

	Min	Median	Q3	Max	Mean	S.D.
Citations-Weighted Patents (CW)	0.00	6.00	17.00	1,560.00	24.51	84.52
Number of Patents (P)	0.00	3.00	9.00	847.00	12.19	40.83
Number of Citations (C)	0.00	2.00	8.00	887.00	11.80	44.54
Number of Inventors	0.00	16.00	37.00	3,814.00	51.33	171.38
Average Team Size	0.00	2.05	2.55	8.00	2.13	0.87
Share of Outside Collaborators	0.00	0.32	0.46	0.88	0.32	0.20
Technology Age	0.00	12.00	14.11	43.00	11.67	4.52
Technology Herfindahl (OST30)	0.05	0.09	0.12	1.00	0.11	0.07
Number of Plants of 200+ Employees	0.00	18.00	29.00	459.00	26.94	44.64
Number of Industry Workers	1,011	19,034	30,164	569,152	29,242	50,380
Employment Diversity (3-digits)	0.92	2.69	2.98	3.55	2.63	0.48
Average Squared Centrality ($\lambda = 0.07, \alpha = 0.5, \beta = 0$)	1.00	1.36	1.57	8.68	1.44	0.39

(b) Correlations.

		1	2	3	4	5	6	7	8	9
1	Number of Inventors (ln)	1								
2	Average Team Size (ln)	0.73	1							
3	Share of Outside Collaborators	0.23	0.53	1						
4	Technology Age (ln)	0.73	0.92	0.31	1					
5	Technology Herfindahl (OST30)	-0.39	-0.27	-0.19	-0.26	1				
6	Number of Plants of 200+ Employees (ln)	0.49	0.17	-0.00	0.18	-0.26	1			
7	Number of Industry Workers (ln)	0.56	0.18	-0.00	0.19	-0.29	0.90	1		
8	Employment Diversity (3-digits)	0.29	0.15	0.13	0.13	-0.29	0.23	0.16	1	
9	Average Squared Centrality ($\lambda = 0.07, \alpha = 0.5, \beta = 0$) (ln)	0.44	0.51	0.54	0.29	-0.18	0.16	0.19	0.15	1

Notes: The explanatory variables based on the network are constructed using a 5-year window (it concerns the following variables: number of inventors, average team size, share of outside collaborators, technology age, technology Herfindahl, and the network-centrality variables). The average squared centrality is computed using the parameters' estimates of the baseline model of Section 6, i.e. in Model (2) of Table 8.

Table 8: Baseline Poisson estimations.

Model:	(1)	(2)	(3)	(4)
Dependent Variable:	CW_{t+1}	CW_{t+1}	P_{t+1}	C_{t+1}
Network Centrality Parameters				
λ (Connectivity)		0.0701***	0.0394***	0.1089***
		(0.0191)	(0.0124)	(0.0338)
α (Synergy)		0.5003***	0.6528**	0.4161***
		(0.1401)	(0.2562)	(0.1368)
β (Rivalry)		0	0	0
		(–)	(–)	(–)
EA-Tech-Specific Variables				
# Inventors (ln)	0.3498***	0.3525***	0.3772***	0.3306***
	(0.03)	(0.0432)	(0.0426)	(0.0536)
Average Team Size (ln)	-0.2667**	-0.3644***	-0.364***	-0.3809***
	(0.0852)	(0.0741)	(0.0617)	(0.1066)
Share of Outside Collaborators	0.2669*	0.2935**	0.3675**	0.2164
	(0.1465)	(0.1364)	(0.1131)	(0.2079)
Technology Age (ln)	-0.0187	0.0197	0.0104	0.0313
	(0.0523)	(0.0485)	(0.0391)	(0.0695)
EA-Specific Variables				
Technology Herfindahl (OST30)	0.9694*	0.8405	1.1689*	0.5026
	(0.5858)	(0.5744)	(0.6045)	(0.6096)
# Plants of 200+ Employees (ln)	-0.092	-0.097	-0.1034	-0.1051
	(0.0714)	(0.072)	(0.0674)	(0.095)
# Workers (ln)	0.4226***	0.4294***	0.4643***	0.3922**
	(0.115)	(0.1095)	(0.1047)	(0.1391)
Employment Diversity (3-digits)	0.0835	0.0688	0.0303	0.0872
	(0.073)	(0.0694)	(0.0744)	(0.079)
Dummies				
EA × Tech	Yes	Yes	Yes	Yes
Time × Tech	Yes	Yes	Yes	Yes
Fit statistics				
Observations	8,910	8,910	8,910	8,838
Adj-pseudo R^2	0.90769	0.90785	0.88348	0.86471
BIC	69,589.527	69,493.238	46,099.191	56,316.964

Notes: Fixed-effects Poisson estimations. The dependent variable is in $t+1$ while the explaining variables based on patent data are built using a 5-year window from $t-4$ to t. The parameters λ, α and β are the ones of the network-centrality, as defined in Equation (3). β has an estimated value equal to its lower bound, implying a non definite standard-error, and is thus treated as a fixed parameter. Clustered standard-errors in parentheses (at the EA-Tech level). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

deviation on average. It turns out that this effect is important: all else being equal, a one-standard-deviation increase in the squared centrality of an EA-Tech implies an increase in future inventive production of 13%. The 95% confidence interval of this effect lies between 4% and 28%.[31] Note that the effects of the other variables remain globally stable.

However, these results do not show how the inventors' connections affect their productivity. To get a sense of the "how" question, we need to look to the other two structural parameters of the centrality measure. First, results strongly support the absence of any rivalry effect. The estimated coefficient of rivalry, $\hat{\beta}$, is at its lower admitted bound, 0. Assuming the theoretical model presented in Section 3 holds, this means that, when an inventor gets new connections, this inflicts no negative externality on his/her current collaborators, controlling for his/her potentially varying research efforts. Stated differently, all else being equal, an inventor's productivity does not decrease when one of his/her partners engages in a new collaboration, assuming that partners (and all others) arbitrarily maintain constant his/her research efforts. Turning now to the estimated value of α, we find a strong synergy effect at play as its coefficient is positive and strongly significant, with $\hat{\alpha} = 0.50$. This means, still assuming the theoretical model holds, that a ten percent increase in the efforts of all direct connections of an inventor would raise the social component of his/her effort productivity by five percent points on average, all other factors and interactions remaining constant. While quite high in our view, that effect is significantly less than unity, as it is assumed to be in standard theoretical models.

To ensure that those results do not rest upon our choice of dependent variable, measured by the number of citations-weighed patents (CW), we run the econometric analysis on two other measures of invention: the number of patents (P) and the number of citations (C). The results of these estimations are reported in Models (3) and (4) of Table 8. For both dependent variables the coefficient of connectivity, λ, is positive and significant. It is lower when the dependent variable is P and higher when the dependent variable is C. However, the overall effects of the network remain similar (unreported but available from the authors). Further, the coefficient of rivalry, β, remains equal to 0. The main difference is that, in Model (3), for patent counts, the synergy coefficient is now equal to 0.65, while in Model (4) this coefficient remains strongly significant, but slightly lower with a value of 0.41. As the coefficient of connectivity is also higher when the dependent variable is the number of citations, at 0.11, this suggests that inter-inventor connections are more beneficial in terms of more quality-orientated measures. All in all, the results are maintained in essence with these two different

[31] Finding the 95% confidence interval is not straightforward because the value of the centrality depends on two variables that are different from zero: λ and α. To obtain the 95% confidence interval, we sampled 1,000 draws of both λ and α along a normal law of mean and standard error their estimated ones (in model (2) of Table 8). For each draw, we computed the average squared centrality for each EA-Tech and its associated standard-deviation. We finally report the bounds (min and max) of the magnitude of the effect of the centrality on urban invention after trimming for the 25 highest and 25 lowest values.

dependent variables. The baseline estimates of Model (4) are, as expected, right between the ones of Model (3) and Model (4). Here again, the effects of the other variables remain globally unchanged with the exception of the "outside collaborators" variable which does not affect the quality of the patents produced locally.[32]

7 Robustness checks

We now propose several robustness checks and extensions. In a nutshell, we i) cope with the reverse causality issue, ii) document and fend off the possible sorting effect of inventors across cities, iii) control for the influence of star inventors, iv) exclude the Paris region, v) change the spatial scale, and vi) reproduce the analysis at the EA level, by pooling all technologies.

7.1 Reverse causality

We consider a first channel of endogeneity that could bias our previous estimates. The urban centrality is calculated using the patterns of collaborations between inventors, measured via patent co-invention. Similarly, the dependent variable is based on patent data. For the sake of argument, assume that centrality does not affect invention, but that centrality values at the city level are determined by the number of patents produced in these cities so that a high centrality value may come from a high number of patents produced. Assume also that, due to temporal auto-correlation, the past number of patents determines future outcomes as measured with the dependent variable.[33] If these three assumptions hold, we would find a spurious positive effect of past centrality on present invention which would only be due to reverse causality and temporal serial correlation of the dependent variable. However, if this is true, then including past levels of the dependent variable in the model would capture the effect of the centrality that stems from this channel of endogeneity.

Therefore, we include the lag of the dependent variable in the regression. The results are provided in Model (1) of Table 9. The lag of the dependent variable has a positive effect on future urban invention and the results regarding the centrality components are remarkably similar to the baseline results: the estimated value of λ is still close to 0.07, the synergy coefficient is slightly reduced to 0.48 and there is still no evidence of rivalry.

We are aware that including the past value of the dependent variable may introduce

[32]Note that we also introduce the estimated centrality measure into a spatial model with a spatial error and a spatial lag of the dependent variable. We find no significant spatial dependence which indicates that the possible spatial bias is limited.The methodology and the results are available from the authors upon request.

[33]Note that the average temporal auto-correlation of the citation-weighted number of patents at the EA-Tech level is of only 10%. We further have tested for the presence of panel unit-roots, the results of the tests are negative.

Table 9: Robustness checks - Endogeneity controls, excluding mobile inventors, star inventors and the Paris region.

Model:	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:	CW_{t+1}	C_{t+1}	CW_{t+1}	C_{t+1}	CW_{t+1}	CW_{t+1}
Other Information:	Endogeneity I	Endogeneity II	Mobile Inventors Are Excluded	Mobile Inventors Excluded Endogeneity II	Star Inv. Are Excluded	Paris Region Excluded
Network Centrality Parameters						
λ (Connectivity)	0.0775***	0.1323***	0.0594***	0.0918***	0.0848***	0.0784***
	(0.0235)	(0.0475)	(0.0166)	(0.0264)	(0.0235)	(0.0216)
α (Synergy)	0.4884***	0.4031***	0.7317***	0.6938***	0.4865***	0.4852***
	(0.1378)	(0.1331)	(0.0905)	(0.0632)	(0.1679)	(0.1448)
β (Rivalry)	0	0	0	0	0	0
	(–)	(–)	(–)	(–)	(–)	(–)
EA-Tech-Specific Variables						
# Inventors (ln)	0.2835***	0.2518***	0.5271***	0.3379***	0.3418***	0.3154***
	(0.0437)	(0.0557)	(0.0572)	(0.0889)	(0.0428)	(0.044)
Average Team Size (ln)	-0.2993***	-0.3082**	-0.4383***	-0.3378*	-0.3516***	-0.289***
	(0.0732)	(0.1073)	(0.133)	(0.1971)	(0.0748)	(0.0763)
Share of Outside Collaborators	0.191	0.0991	0.3077	0.1082	0.2985**	0.1742
	(0.1345)	(0.2101)	(0.2335)	(0.3443)	(0.1389)	(0.1403)
Technology Age (ln)	0.0101	0.0229	0.0752	0.1143	0.015	-0.0101
	(0.0472)	(0.0685)	(0.0829)	(0.1159)	(0.0487)	(0.0497)
EA-Specific Variables						
Technology Herfindahl (OST30)	0.8616	0.5121	1.2805**	1.2004	0.7726	0.9528*
	(0.5697)	(0.5991)	(0.5813)	(0.7878)	(0.5544)	(0.5492)
# Plants of 200+ Employees (ln)	-0.0909	-0.0994	-0.2044*	-0.112	-0.104	-0.1712**
	(0.0706)	(0.0941)	(0.1192)	(0.1548)	(0.0722)	(0.0747)
# Industry Workers (ln)	0.4258***	0.3869**	0.551***	0.4646**	0.4579***	0.3105*
	(0.1085)	(0.1373)	(0.1573)	(0.2127)	(0.1092)	(0.161)
Employment Diversity (3-digits)	0.0641	0.0838	-0.059	-0.0968	0.0511	0.0105
	(0.0683)	(0.0774)	(0.0939)	(0.1309)	(0.0692)	(0.0961)
Other Variables						
Citation-Weighted Patents (ln)	0.0466***					
	(0.0088)					
# Patents (ln)		0.0574***		0.0968***		
		(0.0154)		(0.0288)		
Dummies						
EA × Tech	Yes	Yes	Yes	Yes	Yes	Yes
Time × Tech	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	8,910	8,838	8,568	7,956	8,910	8,280
Adj-pseudo R^2	0.90811	0.86495	0.87638	0.82713	0.90382	0.77962
BIC	69,321.958	56,235.178	52,083.967	39,772.69	68,945.637	61,851.896

Notes: Fixed-effects Poisson estimations. The dependent variable is in $t+1$ while the explaining variables based on patent data are built using a 5-year window from $t-4$ to t. The parameters λ, α and β are the ones of the network-centrality, as defined in Equation (3). β has an estimated value equal to its lower bound, implying a non definite standard-error, and is thus treated as a fixed parameter. Clustered standard-errors in parenthesis (at the EA-Tech level). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

a bias on the coefficients because we are in a panel setup (for a discussion of this issue, see for instance Windmeijer, 2008). To circumvent this problem, we take advantage of the "quantity" (number of patents) and "quality" (citations received) information contained in our data. We thus regress the future "quality" of urban patenting while controlling for the past "quantity" produced. This setup has two advantages: i) the quantity measure is likely to be a better control for the network centrality than the quality,[34] and ii) since quantity and quality variables are of a different nature, the estimated coefficients should not suffer from bias. In Model (2) of Table 9 we can see that the results are also very similar to the ones of Model (4) in Table 8: we find a positive and significant effect of the network (λ) with synergy but no rivalry.

7.2 Dealing with spatial sorting

A second form of endogeneity could arise due to omitted variables. Thought we have a rich set of covariates, still it could be possible that time-varying unobserved variables might affect both present network centrality and future invention. However, as we have shown in the previous sub-section, our results are robust to the introduction of the current production variables. Therefore, to alter our results, an omitted variable should be both correlated with a city's network centrality but orthogonal to its current production. The only such variables we could think of are related to the potential variation of inventors abilities due to spatial sorting of inventors across cities. The urban areas which host the most central inventors may attract the best brains, and thus turn out to be more productive. If so, the interpretation of the results would differ: a high network centrality would imply higher productivity not because of the spillovers and synergies between connected agents but because of its higher attraction power. Moreover, the most productive cities may attract the most productive inventors, who are also the most central in the network. Because of serial correlation in the dependent variable, lagging the dependent variable might not be sufficient to get rid of the bias in this form of endogeneity.

A first way to examine this issue is to study mobility behaviors. For this purpose we build a database of mobilities between EA. We consider that an inventor has moved from EA a to EA a' when he has a patent in EA a in period t and he has a patent in a' in period $t' > t$ and a' being the first different city reported in the subsequent patents of the inventor. The moves signaled by two dates which correspond to the same year are dropped to exclude too-close application dates. As the EPO does not use the invention date as the USPTO but the application date, too-close dates could actually correspond to reversed order invention

[34] For instance, a city can produce 10 patents while receiving no citations, and conversely, a city can receive 100 citations while producing only one patent. It is then clear that as the network is based on patent-collaborations, only the production of patents "makes" the network, independently of the citations they receive.

dates, in which case mobility inference would also be reversed. We find 5,329 such moves between two distinct urban EAs. Table 10 reports the relative differences between the origin and the destination EA-tech characteristics at the time of the last patent in the origin EA (t). Results when the year of the first patent in the EA of destination is used are qualitatively similar. We see that the moves are toward less concentrated areas in numbers of inventors and patents (all measures). The moves are to areas that are very similar in terms of all per capita efficiency measurements as well as in terms of average centrality. So it seems that our results should not be biased by the spatial sorting of inventors.

Table 10: Difference between the destination EA-tech characteristics and the origin EA-tech for all inventors moves.

Relative difference in the	mean	median	std dev	min	max
Number of inventors	-0.234	-0.643	1.457	-2	1.997
Number of patents	-0.238	-0.643	1.46	-2	1.997
Number of citations	-0.234	-0.697	1.495	-2	2
Number of weighted patents	-0.236	-0.658	1.471	-2	1.999
Number of patents per inventor	0.028	0.009	0.467	-1.716	1.871
Number of citations per inventor	0.012	-0.008	0.648	-2	2
Number of weighted patents per inventor	0.024	0.005	0.511	-1.728	1.913
Average squared centrality	-0.004	-0.002	0.079	-0.385	0.340
Total accumulated squared centrality	-0.143	-0.463	1.459	-1.990	1.998

Notes: For each move and each variable considered, the relative difference is computed as $2 \times (destination - origin)/(destination + origin)$. The network centrality (last line) is computed using the baseline estimates of Model (2) of Table 8.

Nevertheless, to complement this descriptive analysis, we re-do the econometric regressions excluding the mobile inventors. The dependent variable is computed afresh without considering their production. Further, the production of the inventors who have their first patent in the period $t+1$ are also excluded, so that the variables are preserved from potential bias due to the arrival of potential newcomers of better quality in some EA-tech. The dependent variable is thus only based on the production of inventors who were already active and, we suspect, have never moved from the city. For consistency, we also exclude the centrality of all mobile inventors in the computation of the EA's average squared network-centrality. The results obtained with this new variable are reported in Model (3) of Table 9, which confirms our previous observation. We see that they are very close to the baseline results in Model (2) of Table 8 though the coefficients of connectivity are slightly lower and the coefficient of synergy is slightly higher. Finally, we show, in Model (4) of Table 9, that the results of Model (2), which controls for endogeneity by using past production as a control, do not differ when we exclude mobile inventors from the sample.

7.3 Controlling for stars

With the type of centrality indexes we are using, the distribution of centrality in the population might be very skewed. There was thus the concern that the results could be driven by star inventors who would also be outliers in terms of centrality. We have implicitly assumed that all inventors' squared centrality matters and can be simply aggregated so that the average squared centrality is considered in the regressions. If only the centrality of stars drives the results, this would partly flaw our assumption. Interpretation of the results would also be affected. To control for such a possible over-influence of stars,[35] we run a new analysis in which the production of EA-techs is regressed on average squared network centrality computed while excluding star inventors.

More precisely, the new centrality variable is obtained in two steps. In the first step, the centrality of all inventors is computed using the whole network, stars included. The difference with the original variable lies in the second step where we average the squared centrality of only non-star inventors at the EA-Tech level. Formally, let $Inv_{a,f,t}^{no\_Star}$ be the set of inventors of the EA-Tech that are not defined as stars. Then the non-star EA-Tech average network centrality is defined as:

$$\overline{\left(c_{a,f,t}^{no\_Star}\left(g_t,\lambda,\alpha,\beta\right)\right)^2} = \frac{1}{\#Inv_{a,f,t}^{no\_Star}} \times \sum_{i \in Inv_{a,f,t}^{no\_Star}} c_{i,t}^2\left(g_t,\lambda,\alpha,\beta\right).$$

Star inventors are defined anew for each year t, based on their production between $t-4$ and t.[36] An inventor is designated as a 'star' if the number of patents he/she produced in a given 5-year window is strictly greater than the top 1% percentile.[37] Those star inventors are then dropped from the population of inventors of their EA-tech as well as their contribution to forward inventions. The results of this estimation are reported in Table 9, Model (5). We find that the estimated connectivity λ and synergy α, are still positive and significant and the estimated rivalry is still equal to 0. The main difference with our baseline results comes from the connectivity coefficient which is higher, reaching a value of 8.4%. These estimations overall mean that the results and the interpretation are hinging on the centrality of all inventors and not only on the few star inventors present in the EA-Tech.

[35] This procedure is intended to control only for their direct influence on urban productivity, and not for their indirect influence, that is, the influence transferred to their neighbors in the network.

[36] The 'average squared centrality' for EA-Techs in which only star-inventors reside is set to 1 (which is the minimum value for this variable), as for EA-Techs in which there is no inventor at all.

[37] The cut-off for being in the top 1% inventors increases gradually from 8 patents in 1985 to 12 patents in 2002.

7.4 The Paris region

The employment areas at least partly located in Ile-de-France, the Paris region, represent a very significant amount of French invention. We are thus interested to see whether these employment areas could drive our results. To check that, we reproduce the baseline model having dropped all yearly observations of the six EA-tech associated with Ile-de-France. This leads to Model (6) of Table 9 which does not exhibit significant changes with respect to the baseline model.

7.5 Different spatial aggregation units

A common concern arising when dealing with discrete geographical units is the moving areal unit problem (MAUP). Because space is continuous and geographical units are discrete in nature, the results can be reliant on the choice of these geographical units. To cope with this issue, the econometric analysis of the baseline model is replicated: i) using NUTS 3 geographical units and ii) constructing and using a more disaggregated geographical unit, that we will name co-invention clusters.

7.5.1 NUTS 3 geographical units

In France, the NUTS 3 regions correspond to the 'départements' which were defined by the French administration. They are larger aggregates than the EAs: continental France is divided in 94 NUTS 3 regions while there are a total of 297 EAs.[38] Since we are interested only in urban areas, we then use the 80 NUTS 3 areas that contain at least one of the 71 urban EAs (remember that EAs are not defined by administrative boundaries and can span different NUTS 3 regions).

The estimates for this geographical unit are reported in Model (1) of Table 12. The results are qualitatively similar to the main results at the EA level. The coefficient of rivalry is still found to be equal to 0, while the synergy and connectivity coefficients are positive with orders of magnitude close to those of the baseline model.

Controlling for R&D Using NUTS 3 areas as spatial aggregation units gives us the opportunity to control for the R&D expenditures realized at this local level. To this end, we make use of the French annual R&D surveys for the period 1985-2002, produced by the French Ministry of Research, on all firms located in France with at least one full time equivalent R&D employee. Those firms have to report the total amount of their R&D expenditures realized in each of the French NUTS3 (this information is therefore not available at the EA level). Then we ran the same regression as Model (1) adding R&D as a new regressor as

[38] The average EA (NUTS 3) surface area is 1,818 (5,745) square kilometers.

presented in Model (2) of Table 12. The introduction of R&D does not change the results whose coefficient turns out to be positive but not significant. It seems that the number of inventors already captures most of the effect of companies' local R&D expenditures.

7.5.2 Co-invention clusters

Since the MAUP grows in severity as the level of geographic aggregation increases, a robustness check should include the use of sub-EA observation units. Thus we recreate geographical units based on the collaboration patterns of inventors, starting from the lowest geographical level available: the cities.[39] The aim is to create aggregate geographical units that are clusters of neighboring cities, such that internal collaborations between inventors within these units is maximized.

The methodology we use to construct the new units, named co-invention clusters (hereafter CC), consists first in merging any contiguous pair of cities that have a number of collaborations between their inventors greater than 4 over the whole period (1981-2003).[40] Then we add to the previously found core clusters all the cities less than 15km away that have at least 2 collaborations with inventors of the clusters. Finally, we keep only the clusters that produce a significant amount of patents over the period (we create two samples: geographical units with more than 50 [resp. 100] patents over the whole period, for all technologies). We end up with a set of 194 (resp. 118) CC.

Figure 4 reports the spatial distribution of these geographical clusters. The black lines demarcate all 297 continental France EAs, and the areas in gray are the 71 urban EAs. Apart from some exceptions (like the Paris region and the south east), the CC are always within the boundaries of the urban EAs, vindicating our previous use of urban-EAs. Only a few geographical clusters are found in non-urban EAs.

Using these new geographical units combined with the patent's technological field, we re-estimate the main model. The results are reported in Table 11. The results are fairly consistent with our main results: we still find no presence of rivalry (coefficient β is still at its lower bound) and the coefficients of connectivity and synergy are still positive and statistically significant – the main difference is that the coefficient of synergy is now greater than the main estimate.

[39] Continental France is divided into 36,568 cities. In comparison, the US has approximately 16,000 cities while it represents almost 15 times the area of France. But working directly at the city level might not be appropriate to locate invention activities and collaboration since it is very common that individuals live and work in different cities (in the patent document, the inventors' private address is mentioned but not the professional one). Note that this is why we initially used the EAs to prevent this issue.

[40] We could not go lower than 4 collaborations because this would have created too large geographical units and would have merged different clusters.

Table 11: Poisson estimation, new geographical units

Dependent Variable:	CW_{t+1}	CW_{t+1}
Area with overall:	>50 patents	>100 patents
Model:	(1)	(2)
Variables		
lambda	0.0626**	0.0599***
	(0.0245)	(0.0159)
alpha	0.6659***	0.7609***
	(0.212)	(0.1183)
beta	0	0
	(–)	(–)
# Inventors (ln)	0.3084***	0.3273***
	(0.0387)	(0.0395)
Average Team Size (ln)	-0.2587***	-0.2911***
	(0.0499)	(0.057)
Share of Outside Collaborators	0.0312	0.0819
	(0.1217)	(0.1325)
Technology Age (ln)	-0.02	-0.0074
	(0.0337)	(0.0386)
Technology Herfindahl (OST30)	-0.153	0.2904
	(0.2754)	(0.4016)
# Plants of 200+ Employees (ln)	-0.0705**	-0.0774*
	(0.0322)	(0.0434)
# Workers (ln)	0.2379**	0.3997***
	(0.0932)	(0.1145)
Employment Diversity (3-digits)	0.6028***	0.4832
	(0.2284)	(0.3485)
Clusters		
Area × Tech	Yes	Yes
Year × Tech	Yes	Yes
Fit statistics		
Observations	20,610	13,950
Adj-pseudo R^2	0.90423	0.91426
Log-Likelihood	-51,950.15	-40,656.67
BIC	116,625.16	90,007.24

Notes: Fixed-effects Poisson estimations. The dependent variable is in $t+1$ while the explaining variables based on patent data are built using a 5-year window from $t-4$ to t. The parameters λ, α and β are the ones of the network-centrality, as defined in Equation (3). β has an estimated value equal to its lower bound, implying a non definite standard-error, and is thus treated as a fixed parameter. Clustered standard-errors in parenthesis (at the Area-Tech level). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

– Maps new aggregates 50 100 groups.pdf

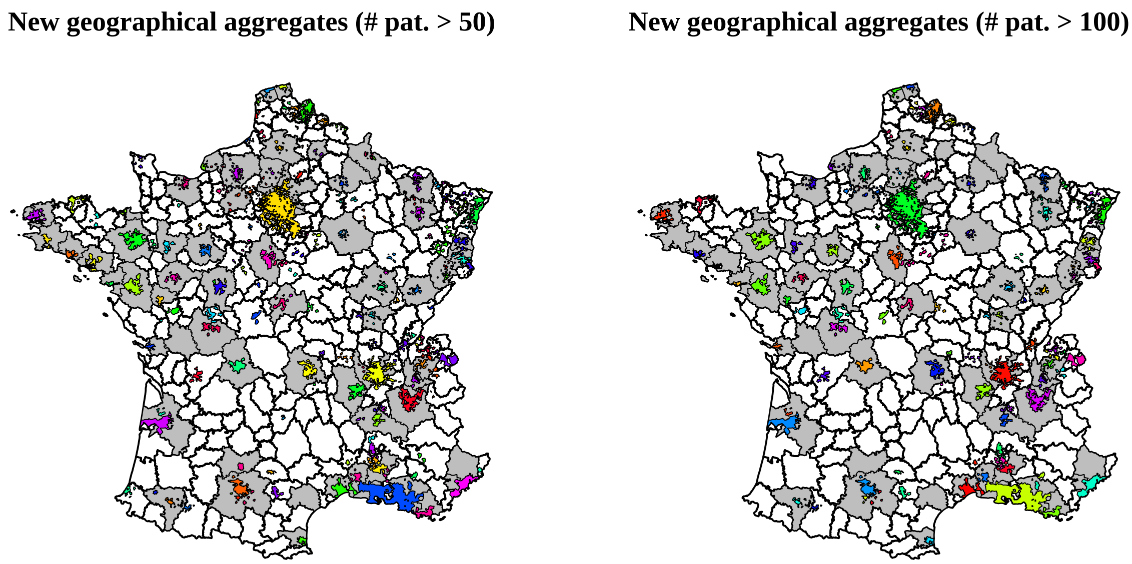


Figure 4: New geographical aggregates obtained from a data driven strategy. Continental France is divided into 297 EAs. The urban EAs are represented in gray. The new geographical aggregates are represented using various colors.

7.6 Without breaking down the data by technologies

Most of the previous studies in the field (Fleming et al., 2007; Lobo and Strumsky, 2008; Breschi and Lenzi, 2016) have conducted their analysis at the MSA level, without breaking down the data by technological fields as we do. We have good reason to do so because it allows us to use a much more refined set of controls. However, here we replicate the analysis at the EA level for comparability to these other studies. As we introduce EA and time fixed effects, the variation now comes only from within-city evolution, without direct controls for the technology developed in the city. Models (3) to (5) of Table 12 present the main results. Model (3) mirrors the baseline model. In Model (4) we focus on the "quality" of the urban production (i.e. the number of citations). Model (5) replicates this last model but including the number of patents as a control. Overall we find similar patterns: positive and significant effect of the network with synergy and no rivalry. However, we observe that the coefficients of connectivity and synergy are above those of the main regressions. This would suggest that omitting technology controls may actually lead to a slight overestimation of the network effects.

Table 12: Robustness checks and extensions - Regressions at the EA level and when the centrality is not squared.

Model:	(1)	(2)	(3)	(4)	(5)
Dependent Variable:	CW_{t+1}	CW_{t+1}	CW_{t+1}	C_{t+1}	C_{t+1}
Other information:	Geo. Level of Analysis NUTS 3	Control for R&D Analysis NUTS 3	Urban EA Baseline	Urban EA Citations	Urban EA Endogeneity II
Network Centrality Parameters					
λ (Connectivity)	0.0561***	0.0558***	0.0858***	0.1384***	0.1757***
	(0.0163)	(0.0162)	(0.0225)	(0.033)	(0.0434)
α (Synergy)	0.6137***	0.6242***	0.6126***	0.4843***	0.4585***
	(0.1391)	(0.138)	(0.0984)	(0.0829)	(0.088)
β (Rivalry)	0	0	0	0	0
	(–)	(–)	(–)	(–)	(–)
EA-Tech-Specific Variables					
(EA-specific for models 3 to 5)					
# Inventors (ln)	0.3843***	0.3757***	0.6237***	0.5425***	0.3421***
	(0.0351)	(0.0347)	(0.0641)	(0.0802)	(0.103)
Average Team Size (ln)	-0.3603***	-0.3553***	-0.7629**	-0.72	-0.4612
	(0.0795)	(0.0784)	(0.3585)	(0.4972)	(0.4886)
Share of Outside Collaborators	0.0375	0.0343	0.779*	0.6764	0.403
	(0.1271)	(0.1265)	(0.455)	(0.6347)	(0.6072)
Technology Age (ln)	0.007	0.0093	-0.3691**	-0.3804*	-0.3112
	(0.0514)	(0.0514)	(0.1507)	(0.2088)	(0.2082)
EA-Specific Variables					
Technology Herfindahl (OST30)	1.2145**	1.2229**	0.6387	0.3322	0.1652
	(0.5352)	(0.5272)	(0.5415)	(0.5853)	(0.5451)
# Plants of 200+ Employees (ln)	-0.0096	-0.0167	-0.0557	-0.0749	-0.0474
	(0.087)	(0.0863)	(0.0583)	(0.0866)	(0.0841)
# Industry Workers (ln)	0.2729**	0.2198*	0.2661***	0.2508**	0.2065*
	(0.1231)	(0.126)	(0.0764)	(0.1224)	(0.12)
Employment Diversity (3-digits)	-0.0622	-0.0619	0.0257	0.0654	0.0676
	(0.0487)	(0.0484)	(0.0663)	(0.0882)	(0.0845)
Other Variables					
# Patents (ln)					0.2021***
					(0.0613)
R&D expenditures (ln) (NUTS 3)		0.0467			
		(0.0452)			
Dummies					
EA	No	No	Yes	Yes	Yes
EA×Tech	Yes	Yes	No	No	No
Time	No	No	Yes	Yes	Yes
Time×Tech	Yes	Yes	No	No	No
Fit statistics					
Observations	10,080	10,080	1,278	1,278	1,278
Adj-pseudo R^2	0.8814	0.88142	0.97448	0.95659	0.95684
BIC	84,762.868	84,755.058	15,014.114	13,406.488	13,337.704

Notes: Fixed-effects Poisson estimations. The dependent variable is in $t+1$ while the explaining variables based on patent data are built using a 5-year window from $t-4$ to t. The parameters λ, α and β are the ones of the network-centrality, as defined in Equation (3). β has an estimated value equal to its lower bound, implying a non definite standard-error, and is thus treated as a fixed parameter. Clustered standard-errors in parenthesis (at the EA-Tech level for models 1-2 and at the EA level for models 3-5). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

8 Endogenous network formation

In this section we deal specifically with the possible endogeneity of network formation. The fixed-effects setup we employed in previous regressions significantly reduces the endogeneity issues as it controls for all potential confounding factors that would be specific to the EA-Tech or to the Year-Tech. Inventor centrality would, however, be endogenous to future invention if unobserved time-varying EA-Tech specific factors do affect both network formation and inventive performance. For instance, previous collaborations or other dyadic variables may influence both the formation of links and agent productivity. The estimated coefficient of average squared centrality indexes would thus be biased, as it would convey the effect of those omitted variables.

We would like network statistics to be "free" of those effects that cannot be controlled for in the main regression. To test whether the effect of centrality on invention is robust to such form of endogeneity, we introduce, like König et al. (2014), a preliminary stage in which a network of collaboration is generated. This network is generated so that it mimics the main characteristics of the observed network but is explained only by determinants that will be controlled for in our final estimation. This guarantees that agent centrality in this network is not correlated to omitted variables, conditional on the explaining variables. Appendix A.6 describes in more detail how the network is generated. We then apply a standard instrumental variable procedure. We regress the observed average squared centrality on the average squared centrality of inventors in the generated network. Invention performance can then be regressed either on the observed network centrality controlling for the first stage residuals, or directly on the predicted average squared centrality, which is freed of any correlations with potentially omitted variables.

Note that, due to their non-linear nature, the three centrality parameters (λ, α and β) cannot be estimated in this setup. We can however verify that the previously estimated effect of the average squared network centrality is robust to network endogeneity. The average squared centrality is thus computed using the estimated parameters of Model (2) of Table 8, i.e. $\hat{\lambda} = 0.07$, $\hat{\alpha} = 0.50$ and $\hat{\beta} = 0$.

Table 13 presents the results. In Model (1), we check that the estimated elasticity of the average squared centrality variable is close to the estimated elasticity of the number of inventors (as suggested in Equation 10). This is verified as the estimated coefficients are very close (.38 and .35). The first stage equation of the IV is presented in Model (2). As expected, the average squared centrality of the network generated in the preliminary stage is strongly correlated with the average squared centrality of the observed network (with a coefficient of 0.41 and an F statistic of 1401). Second stage estimations are presented in the two last columns. In Model (3) we adopt a control function approach: we insert the residual of the first stage regression into the main estimation (see Terza et al., 2008; Wooldridge, 2014). In Model (4), we report regression coefficients when the average squared centrality variable is replaced by the first stage predictions. Terza et al. (2008) argue that this latter approach is less consistent than the control function in nonlinear models. The results are very similar, however. They clearly show that the average squared centrality of inventors estimated previously (Model 2 of Table 8) remains a positive and strongly significant predictor of future invention of the EA-Techs.

9 Are social connections mediated by geographical distance?

So far, we have considered that the benefits of network connections do not depend on geography: ceteris paribus, a collaboration with a geographically distant partner has the same benefit as a collaboration with an inventor located next door. Though this sounds like a natural assumption to make in the network literature (see Jackson and Wolinsky 1996; Carayol and Roux 2009; Carayol et al. 2015), it sounds much less natural in economic geography where the effectiveness of social relations is often considered to be affected by distance. In this section, we relax this assumption so that the benefits of network connections may now vary with geographical distance. For that purpose, we modify the social component of inventor productivity previously defined in Equation (2), as follows:

$$\psi_i\left(g^c, g^d, \lambda^c, \lambda^d, \alpha, \beta\right) \equiv \lambda^c \sum_j g^c_{ij} e^{\alpha}_j d_j^{-\beta} + \lambda^d \sum_j g^d_{ij} e^{\alpha}_j d_j^{-\beta}, \tag{12}$$

where g^c_{ij} (resp. g^d_{ij}) is non null and equal to unity for all pairs of connected inventors (i and j) being distant from less (resp. more) than a given threshold. As we allow agents to benefit from their close and distant partners with different strengths (λ^c and λ^d), inventors exert equilibrium efforts equal to a composite network centrality,[41] given by:

$$c_i\left(g^c, g^d, \lambda^c, \lambda^d, \alpha, \beta\right) = 1 + \lambda^c \sum_j g^c_{ij} c^{\alpha}_j d_j^{-\beta} + \lambda^d \sum_j g^d_{ij} c^{\alpha}_j d_j^{-\beta}. \tag{13}$$

Then we re-estimate the main model, but using Equation (13) to obtain the centrality statistics in Equation (4). From there, all further calculations remain identical. The results are displayed in Table 14. We test three different thresholds: 30 km, 50 km and 70 km. Geographic distances are computed using "as the crow flies" distances in kilometers between i and j inventors' city centroids. Note that the mean geographic distance between centroids of all pairs of cities in an EA is about 43 km.

[41] This network centrality is also positive and unique. The proof is similar to that in Appendix 10.

Table 13: Instrumental variables regressions, using the inventors' centrality on a generated network as an instrument.

Model:	(1)	(2)	(3)	(4)
Dependent Variables:	CW_{t+1}	$\ln\overline{\left(c_{a,f,t}\left(g_t,\hat{\lambda},\hat{\alpha},\hat{\beta}\right)\right)^2}$	CW_{t+1}	CW_{t+1}
Method:	*Poisson*	*OLS*	*Poisson*	*Poisson*
Estimation:	Normal	1^{st} stage	2^{nd} Stage Control Function	2^{nd} Stage Predicted Variable
Centrality				
$\ln\overline{\left(c_{a,f,t}\left(g_t,\lambda=0.07,\alpha=0.50,\beta=0\right)\right)^2}$	0.3807***		1.6224***	
	(0.1182)		(0.3352)	
IV related variables				
$\ln\overline{\left(c_{a,f,t}\left(g_t^{Gen.},\lambda=0.07,\alpha=0.50,\beta=0\right)\right)^2}$		0.4074***		
		(0.0332)		
Residual 1^{st} Stage			-1.3599***	
			(0.3526)	
Predicted Av. Sq. Centrality 1^{st} Stage (ln)				1.6379***
				(0.3362)
EA-Tech-Specific Variables				
# Inventors (ln)	0.3494***	0.003	0.3008***	0.291***
	(0.0437)	(0.0077)	(0.0404)	(0.0407)
Average Team Size (ln)	-0.3688***	0.1538***	-0.5398***	-0.5108***
	(0.0755)	(0.0175)	(0.0847)	(0.0837)
Share of Outside Collaborators	0.2907**	0.1944***	0.0152	-0.0562
	(0.1365)	(0.0303)	(0.1595)	(0.1621)
Technology Age (ln)	0.023	-0.1025***	0.15***	0.1499***
	(0.0496)	(0.0111)	(0.0511)	(0.0509)
EA-Specific Variables				
Technology Herfindahl (OST30)	0.8338	0.0802	0.6801	0.7341
	(0.5762)	(0.0635)	(0.5304)	(0.5255)
# Plants of 200+ Employees (ln)	-0.0976	0.0135	-0.1217*	-0.1231*
	(0.0721)	(0.0085)	(0.0702)	(0.071)
# Workers (ln)	0.4321***	0.0072	0.4484***	0.4459***
	(0.1102)	(0.0363)	(0.1055)	(0.1048)
Employment Diversity (3-digits)	0.0675	-0.0128	0.0892	0.1036
	(0.0696)	(0.0157)	(0.0689)	(0.0687)
Dummies				
EA × Tech	Yes	Yes	Yes	Yes
Year × Tech	Yes	Yes	Yes	Yes
Fit statistics				
F-stat		1401.376		
Observations	8,910	8,910	8,910	8,910
Adj-pseudo R^2 (R^2 for model 2)	0.90967	0.7914	0.90971	0.90961
Log-Likelihood	-31,864.77	15,551.82	-31,849.72	-31,886.33
BIC	69,450.25	-25,382.93	69,429.25	69,493.37

Notes: The dependent variable is in $t+1$ while the explaining variables based on patent data are built using a 5-year window from $t-4$ to t. The parameters λ, α and β are the ones of the network-centrality, as defined in Equation (3). Clustered standard-errors in parenthesis (at the EA-Tech level). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

This allows us to obtain further interesting results. First, the benefits from both close and distant collaborations remain positive and significant in all Models (1)-(3). Second, the connectivity effect for close collaborations is always greater than the connectivity effect from more distant collaborations ($\hat{\lambda}^c > \hat{\lambda}^d$). This suggests that connections are more beneficial when formed between close individuals. Third, the benefits from more distant connections decrease significantly with the distance threshold. When a more distant connection is defined as a connection between two persons located more than 70km away from each other (Model 3), the benefit from such partnership is only 60% that of a connection a shorter distance away.

Table 14: Poisson estimation with differentiated effects from collaborating at a distance lower or greater than 30km (resp. 50km and 70km).

Dependent Variable:	CW_{t+1}	CW_{t+1}	CW_{t+1}
Close connections defined as:	$< 30km$	$< 50km$	$< 70km$
Model:	(1)	(2)	(3)
Network Centrality Parameters			
λ^c (Connectivity, close)	0.0706**	0.0711**	0.0742***
	(0.0305)	(0.0296)	(0.0248)
λ^d (Connectivity, distant)	0.0688***	0.0667***	0.0446**
	(0.0239)	(0.0221)	(0.019)
α (Synergy)	0.506***	0.5146***	0.6211***
	(0.1394)	(0.1397)	(0.1241)
β (Rivalry)	0	0	0
	(–)	(–)	(–)
EA-Tech Specific Variables			
# Inventors (ln)	0.3523***	0.3521***	0.3494***
	(0.0428)	(0.0434)	(0.0434)
Average Team Size (ln)	-0.3648***	-0.3653***	-0.3696***
	(0.0773)	(0.0769)	(0.076)
Share of Outside Collaborators	0.2974**	0.3023**	0.3528**
	(0.1433)	(0.1408)	(0.1401)
Technology Age (ln)	0.02	0.0204	0.0238
	(0.0495)	(0.0497)	(0.0494)
EA Specific Variables			
Technology Herfindahl (OST30)	0.8391	0.8374	0.8183
	(0.5673)	(0.573)	(0.5717)
# Plants of 200+ Employees (ln)	-0.0971	-0.0973	-0.0987
	(0.0722)	(0.0723)	(0.0722)
# Workers (ln)	0.4298***	0.4303***	0.4364***
	(0.1086)	(0.1091)	(0.1091)
Employment Diversity (3-digits)	0.069	0.069	0.0691
	(0.0704)	(0.0698)	(0.0698)
Dummies			
EA × Tech	Yes	Yes	Yes
Time × Tech	Yes	Yes	Yes
Fit statistics			
Observations	8,910	8,910	8,910
Adj-pseudo R^2	0.90965	0.90965	0.90965
Log-Likelihood	-31,872.61	-31,872.57	-31,870.35
BIC	69,493.22	69,493.15	69,488.70

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Notes: Fixed-effects Poisson estimations. The dependent variable is in $t+1$ while the explaining variables based on patent data are built using a 5-year window from $t-4$ to t. The parameters λ^c, λ^d, α and β are the ones of the network-centrality, as defined in Equation (13). β has an estimated value equal to its lower bound, implying a non definite standard-error, and is thus treated as a fixed parameter. Clustered standard-errors in parenthesis (at the EA-Tech level). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

10 Conclusion

In this paper, we re-examine the role of collaborative networks between inventors in urban invention. This role has been challenged in the recent literature, which has sought to clarify the effects of agglomeration vs. the effects of networks on the inventive performance of cities. We first introduce a stylized model linking an inventor's productivity to his/her network connections, associating inventors' productivity to the square of their centrality in the network. Our centrality is generic so that the network can presumably affect the inventive performance in different ways. We then test which premises on the way agents affect their neighbors' research productivity best predict future inventions in cities, while controlling for various agglomerative features of cities.

Our results suggest that network connections clearly matter for urban invention. Cities benefit from their inventors' networks in a way which suggests that inventors' productivity increases with their partners' efforts (synergy effect) in a non-rival manner. That is to say, agents' productivity is improved thanks to their partners' efforts without being impaired by other inventors jointly benefiting from those partners. Non-rivalry of connections and synergy effect could provide a new view on interactions between knowledge workers. Improved productivity seems not to come from knowledge that would simply spill over, flowing through connections in a passive way. Rather, it would result from interacting with partners in professional networks that may involve the exchange, confrontation and enrichment of ideas which increase partners' productivity. From a methodological point of view, our results also indicate that the effects of existing centrality measures such as Degree or Katz-Bonacich should be treated cautiously as they may hide some more complex realities on how networks matter. Our results also highlight that geographic distance between inventors alter the strength of their collaboration: connections are more beneficial when they are formed between close individuals.

In terms of policy, the conclusions of the paper support local public policies aiming at increasing connectivity of inventors (within and outside cities) and at attracting locally central agents. The role of star inventors, emphasized in many studies since Zucker et al. (1998), is here highlighted on the grounds of their role in networks. Central agents are beneficial to, and benefit from, their numerous collaborators' productivity without rivalry, and that effect propagates in the network. Of course, these effects have been tested on French inventors only. Natural extensions include applications to other countries, geographical scales, or contexts.

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Appendix

A.1 A more general form of the model

This section follows the model developed in Section 3.1 and shows that the introduction of new parameters in this model imply no significant change to the results.

Consider the following new productivity and utility functions:

$$y_i = e_i \left(\gamma_1 + \psi_i \left(g, e_{-i}\right)\right),$$

$$u_i = e_i \left(\gamma_1 + \psi_i \left(g, e_{-i}\right)\right) - \frac{\gamma_2}{2} e_i^2,$$

where γ_1 is the inventor's own productivity without any collaborator and γ_2 is a parameter scaling the disutility of the effort. Those modifications imply no significant change to the result.

Indeed, with those new parameters, the equilibrium effort of each inventor, $e_i^* \left(g, \lambda, \alpha, \beta, \gamma_1, \gamma_2\right)$, must respect the following system of equations:

$$e_i^* = \frac{\gamma_1}{\gamma_2} + \frac{\lambda}{\gamma_2} \sum_j g_{ij} \left(e_j^*\right)^\alpha d_j^{-\beta}, \ \forall i.$$

Denoting $\gamma = \gamma_1 / \gamma_2$ and dividing by γ yields:

$$\frac{e_i^*}{\gamma} = 1 + \frac{1}{\gamma} \times \frac{\lambda}{\gamma_2} \sum_j g_{ij} \left(e_j^*\right)^\alpha d_j^{-\beta},$$

$$\Leftrightarrow \frac{e_i^*}{\gamma} = 1 + \gamma^{-(1-\alpha)} \frac{\lambda}{\gamma_2} \sum_j g_{ij} \left(\frac{e_j^*}{\gamma}\right)^\alpha d_j^{-\beta}.$$

Note that by writing $\tilde{e}_i^* \equiv e_i^* / \gamma$, then $\tilde{e}_i^*$ respects the centrality defined by Equation (3) and thus can be written as $\tilde{e}_i^* = c_i \left(g, \gamma^{-(1-\alpha)} \lambda / \gamma_2, \alpha, \beta\right)$. This shows that we have the following equivalence:

$$e_i^* \left(g, \lambda, \alpha, \beta, \gamma_1, \gamma_2\right) = \gamma c_i \left(g, \gamma^{-(1-\alpha)} \frac{\lambda}{\gamma_2}, \alpha, \beta\right).$$

Thus including the two parameters, γ_1 and γ_2, to the productivity and the utility functions would merely lead to the introduction of a proportionality coefficient to the centrality measure at equilibrium without providing any distributional change.

A.2 Existence and unicity of the centrality when $\alpha = 1$

When $\alpha = 1$, the centrality has a closed-form and can be obtained as:

$$\mathbf{c}(g,\lambda,\alpha=1,\beta)=(I-\lambda\tilde{g}(\beta))^{-1}\mathbf{1}, \tag{14}$$

where $\mathbf{1}$ is a n vector of ones and $\tilde{g}(\beta)$ is the matrix of typical element $\tilde{g}_{ij}(\beta)\equiv g_{ij}/d_j^{\beta}$ if g_{ij}=1 (which ensures that $d_j>0$) and zero otherwise. Note that $\tilde{g}(\beta)$ is equal to the adjacency matrix if $\beta=0$, and to the column standardized adjacency matrix if $\beta=1$. $\mathbf{c}(g,\lambda,\alpha=1,\beta)$ is the vector of all centralities.

The solution of the system of linear equations characterized by Equation (14) does not generically admit a positive solution when λ is greater than or equal to the inverse of the largest eigenvalue of the matrix $\tilde{g}(\beta)$. In our model, efforts make sense only if positive, and thus so do the centralities. The matrix $(I-\lambda\tilde{g}(\beta))$ is invertible, and its inverse matrix is unique and non-negative (implying that its product with $\mathbf{1}$ is positive) if $\lambda<1/s(\beta)$ where $s(\beta)$ is the largest eigenvalue of the non-negative matrix $\tilde{g}(\beta)$ (this matrix contains only positive elements).[42] Such condition on $\tilde{g}(\beta)$ implies that equilibrium efforts cannot be calculated for any network, connectivity and rivalry.

A.3 Existence and unicity of the centrality when $\alpha\in[0;1[$

In this section we demonstrate Theorem 1. As when $\lambda=0$ and when $\alpha=0$ the proof is trivial, in what follows we consider only $\lambda>0$ and $\alpha\in]0,1[$. We essentially show that the properties of Equation (3) allows us to apply theorems introduced by Kennan (2001) which establishes that there is one and only one positive solution.

Let the function $f:\ \mathbb{R}_+^n\rightarrow\mathbb{R}_+^n$ be given by:

$$f(x)=\mathbf{1}+\lambda Gx^{\alpha},$$

where $x\in\mathbb{R}_+^n$, $\mathbf{1}$ is a n-vector of ones, $\lambda>0$ and $\alpha\in]0;1[$ are fixed scalars, and G is a $n\times n$ matrix of typical element G_{ij} such that $G_{ij}\geq 0$, $\forall i,j$. The vector x^{α} is defined as the vector whose ith element is given by $(x^{\alpha})_i=x_i^{\alpha}$. If we replace G by $\tilde{g}(\beta)$ as defined in A.2, we have exactly Equation (14).

The function f is increasing as its first derivatives, given by

$$\frac{\partial f_i(x)}{\partial x_j}=\alpha\lambda G_{ij}x_j^{-(1-\alpha)},\ \forall i,j,$$

are positive.

Let $\hat{g}\equiv\max\left\{G_{ij}/(i,j)\in\{1,\dots n\}^2\right\}$. Since $\alpha\in]0,1[$, there exists an $\hat{x}\in\mathbb{R}_+$ such that $\hat{x}>1+\lambda\hat{g}\hat{x}^{\alpha}$ (note that $\lim_{\hat{x}\rightarrow+\infty}\hat{x}/(1+\lambda\hat{g}\hat{x}^{\alpha})=+\infty$). Let $\hat{X}$ be the $\mathbb{R}_+^n$ vector such that

[42] See theorem III* of Debreu and Herstein (1953) for a formal proof.

$\hat{X}_i = \hat{x}, \forall i$. We thus have $f\left(\hat{X}\right) = 1 + \lambda G_{ij}\hat{X}^{\alpha} \leq 1 + \lambda \hat{g}\hat{X}^{\alpha} < \hat{X}$. As we also have $f(\mathbf{0}) > \mathbf{0}$, we can thus apply Theorem 3.2 of Kennan, 2001 (implied by Tarski's theorem) which states that if f is increasing, if for a positive vector a, $f(a) > a$ and if $f(b) < b$ for some vector $b > a$, then f has a positive fixed point. We now turn to unicity.

Let function h be defined by $h(x) = f(x) - x$. Assume further that x_f is a fixed point of function f so that $f\left(x_f\right) = x_f$ and $h\left(x_f\right) = 0$. For any δ such that $0 < \delta < 1$, we have $f\left(\delta x_f\right) = \mathbf{1} + \lambda G\left(\delta x_f\right)^{\alpha} = \mathbf{1} + \delta^{\alpha}\lambda G x_f^{\alpha} = \mathbf{1} + \delta^{\alpha}\left(f\left(x_f\right) - \mathbf{1}\right) = \mathbf{1} + \delta^{\alpha}\left(x_f - \mathbf{1}\right)$. As $\alpha \in]0,1[$, we have $\delta^{\alpha} > \delta$ and $\delta^{\alpha} < 1$, and thus $\mathbf{1} + \delta^{\alpha}\left(x_f - \mathbf{1}\right) > \delta x_f$. Therefore $h\left(\delta x_f\right) > 0$. This means h is strictly R-concave. We can thus apply Corollary 3.1 of Kennan (2001) which states that if function f is quasi-increasing and h is strictly R-concave, then there is at most one positive fixed point of f. As we have shown this fixed point exists, it is then also unique. This ends the proof of Theorem 1.$\square$

A.4 Code used to run the estimation example (*R* software)

Finally we provide the code used for the simple example of this section: from the construction of the network, via the calculus of the centrality, to the estimation of the parameters. This code is written in the language of the free statistical software R.[43]

```
# Defining the network
# The matrix G represents the adjacency matrix of the network depicted in Figure 1
G = matrix(0, 7, 7) # an empty 7x7 matrix
# We add the 10 edges of the network
G[1,2] = G[1,5] = G[1,6] = G[1,7] = G[2,3] = G[2,4] = G[2,5] = G[3,4] = G[3,5] = G[4,5] = 1
G = G + t(G) # G is symmetric, we add its transpose
# Defining the function to get the network centrality
abCentrality = function(G, lambda, alpha, beta){

    n = nrow(G)
    # We compute the matrix G_tilde(beta)
    degree = pmax(rowSums(G), 1)
    G_d = t(G/(degree^beta))
    # We start at the vector of 1, and iterate until convergence (ie reaching a fixed point)
    C_old = rep(1, n)
    iterMax = 1000 # Just a control to avoid possible problems
    iter = 0
    maxDiff = Inf # The algorithm stops when maxDiff < 0.0001
    while(maxDiff>1e-4 & (iter <- iter+1)<iterMax){

        C_new = 1 + lambda * G_d %*% C_old^alpha
        maxDiff = max(abs(C_new - C_old))
        C_old = C_new
    }
    if(iter == iterMax) stop("Error: Maximum iterations reached.")
    return(C_new)
```

[43]The code used in the text is more complex since the centrality of each inventor is aggregated at the EA-Tech level. However, this lengthier code used to get the paper's results is available on request.

```
}
# Defining the centrality function that will be used in the optimization process:
centFun = function(alpha, beta, lambda){

      # G is a global variable (i.e. should be loaded in the global environment)
      cent = abCentrality(G, lambda = lambda, alpha = alpha, beta = beta)
      return(cent**2) # the squared centrality is returned

}
# Defining the outcomes (i.e. the dependent variable of the regression)
# There is 7 nodes in the network => we create the vector containing the outcome of each of the nodes
# As in Table 8
myData = data.frame(outcome_1 = c(5, 3.5, 3, 2, 3, 1, 0),
                    outcome_2 = c(2.5, 5, 5, 3, 6, 1, 0),
                    outcome_3 = c(5, 7, 3, 4, 8, 1, 2),
                    outcome_4 = c(5, 4, 2, 1, 3, 6, 7))
# Running the regression (note that we bound the values of lambda, alpha and beta)
# (abbreviation: 'NL' means non-linear)
# install.packages("FENmlm") # installation of the package if necessary
# help(femlm) # Documentation of the function femlm
library(FENmlm)
res = femlm(outcome_1 ~ 1, # The dep. var. and the intercept
             myData, # The data

             NL.fml = ~log(centFun(alpha, beta, lambda)), # The non linear part
             start = list(lambda=0.1, alpha=0.2, beta=0.3), # Starting values of the NL part
             lower = list(lambda = 0, alpha = 0, beta = 0), # Lower bounds of the NL parameters
             upper = list(alpha = 0.99, beta=1)) # Upper bounds of the NL parameters
# We look at the results
summary(res)
# For the results for other outcomes, just change the name of the dependent variable
```

A.5 Varying values of the elasticity

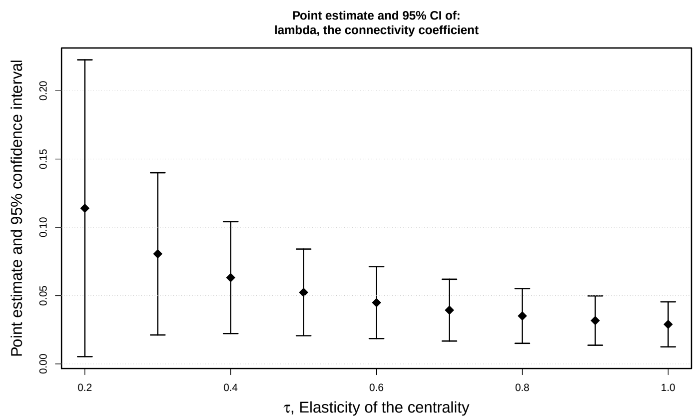
The empirical model of Section 4 assumes the same elasticity, τ, between the number of inventors and the EA-Tech average squared centrality. In this section, we relax this assumption and re-do the baseline econometric analysis, Model (2) of Table 8, for varying elasticities of the average squared centrality. More specifically, instead of Equation (10), we estimate the following equation:

$$E\left(Y_{a,f,t+1}\right) = d_{a,f} \cdot d_{f,t} \cdot \prod_k X_{k,a,f,t}^{\theta_k} \cdot Inv_{a,f,t}^{\gamma} \cdot \overline{c_{a,f}^2\left(g_t, \lambda, \alpha, \beta\right)}^{\tau},$$

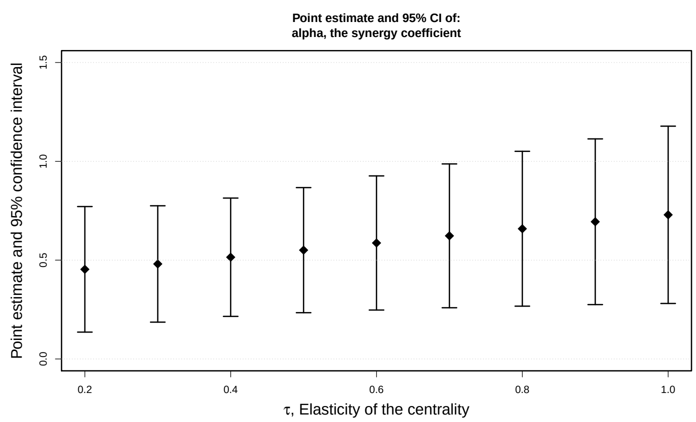
assuming a value of the elasticity τ ranging from 0.2 to 1 by 0.1 increments. In all 8 estimations, we find a rivalry coefficient at its lower bound, 0, similarly to the main results. Figure 5 summarizes the other results. As clearly shown in the figure, we see that the connectivity coefficient and the synergy coefficients remain positive and statistically significant. Looking at the overall effect of the centrality variable on future invention, we also see that the magnitude of the effect and its 95% confidence interval remain fairly stable despite the variation in the elasticity. Therefore, our main results, namely positive effect of the network on urban invention with synergy and no rivalry, are not altered when relaxing this assumption.

Figure 5: Consequence of changing the elasticity of the city average squared centrality.

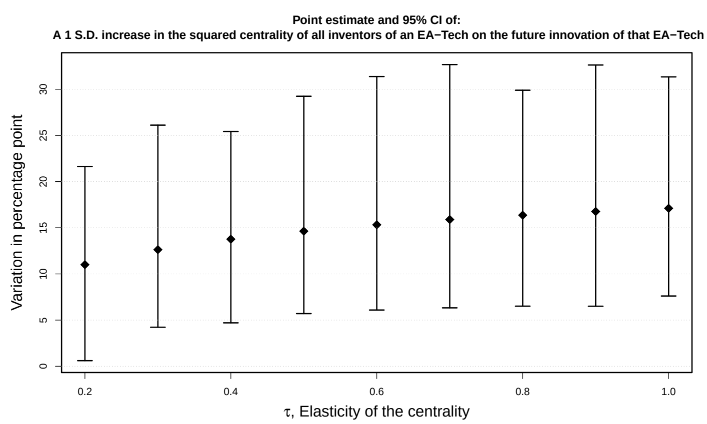
(a) Coefficient and 95% confidence interval estimates of connectivity (λ) for varying values of the elasticity of the average squared centrality variable.



(b) Coefficient and 95% confidence interval estimates of connectivity (λ) for varying values of the elasticity of the average squared centrality variable.



(c) Increase in future urban invention due to a one standard deviation increase in the network-centrality of all inventors of an EA-Tech, for varying values of the elasticity of the average squared centrality variable.



A.6 Generating a network

We here describe the preliminary stage of IV regressions presented in Section 8. The objective is to generate a network relying on observable information that will be later controlled for in the estimation.[44] It turns out that the average squared centralities calculated on the observed and on the predicted networks are very correlated (see first stage regression).

In a fist step, we regress the number of collaborations between each dyad of EA of the same technological field.[45] Although collaborations are undirected, this estimation is very similar to the estimation of trade flows between areas using gravity models (Anderson, 2011). The empirical gravity collaboration equation is:

$$E\left(Collab_{IJ}^{f,t}\right) \;=\; \alpha_{I,f} \times \alpha_{J,f} \times \gamma_{f,t} \times \frac{\prod_k \left(M_{I,f,t,k}^{\beta_{1,k}} M_{J,f,t,k}^{\beta_{2,k}}\right)}{\prod_{k'} D_{IJ,f,t,k'}^{\beta_{3,k'}}} \qquad (15)$$

where the subscript I and J represent the origin and destination EAs, while indexes f is the technological field and t is the year. The dependent variable $Collab_{IJ}^{f,t}$ is the number of collaborations between I and J in tech f and year t. The coefficients $\alpha_{I,f}$ (resp. $\alpha_{J,f}$) are EA-Tech dummies. The variables $M_{I,f,t,k}$ represent masses of attraction between the two EA-Techs. Since the collaboration linkages are undirected, symmetry must hold implying $\beta_{1,k} = \beta_{2,k}$. Consequently, after taking the logs on both side of Equation (15), we use the sums $\left(M_{I,f,t,k} + M_{J,f,t,k}\right)$ as regressors. Time varying masses variables include the number of inventors, the number of industrial workers and the number of large plants in the EA-Techs. The variables $D_{IJ,f,t,k'}$ account for distances between the two EA-Techs. We use a quadratic function of the logarithm of the geographic distance between the two EAs (the distance within an EA, i.e. $distance_{II}$, is set to 1); a dummy taking value 1 for intra-EA flows (i.e. when $I = J$); and a dummy taking value 1 if the two EAs are geographic neighbors.

Equation (15) is estimated with a Poisson model (Santos Silva and Tenreyro, 2006) whose results are reported in Table 15. Geographic distance plays a negative role, and neighboring EAs tend to collaborate more as compared to non neighboring ones. The coefficient for collaborations within the EA is negative but this is compensated by the fact that the geographic distance is equal to 0 within EAs.[46] The number of inventors has a large and

[44] So, we specifically avoid using micro-level determinants that, even if they may influence the formation of collaborations, may as well affect future invention. For instance, past collaborations, common collaborators may both affect the formation of networks and inventors future productivity. Note that including those variables do not affect significantly our IV results.

[45] The technological classes we use (OST 7 classification) are broad enough so that each patent is attached to only one technology. Thus, there is by definition no collaboration between EAs of different technologies.

[46] The negative effect of geographic distance is equal to the "Within EA" coefficient (-7.24) at a distance of 8 km which means that the "Within EA" is always greater than the negative effect of distance.

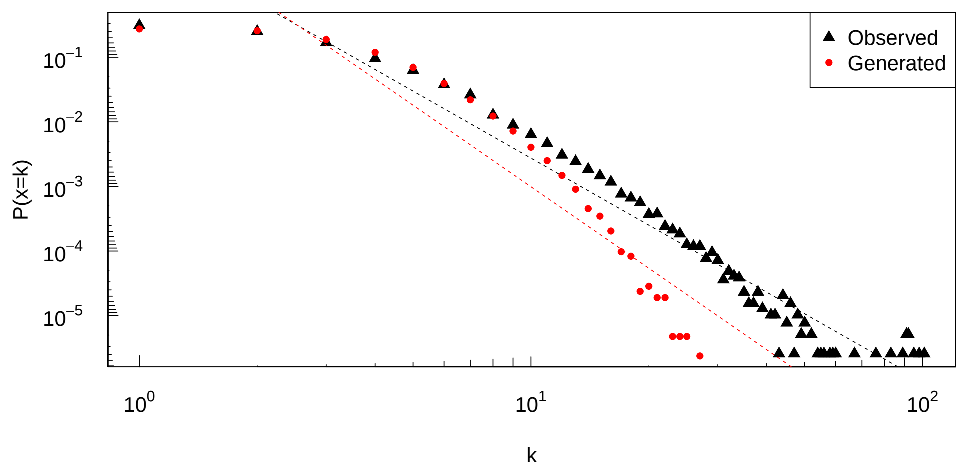
significant elasticity.

Table 15: Gravity estimation of collaboration flows between EA-Techs using a Poisson model.

Model:	(1)
Dependent Variable:	$Collab_{I,J}^{f,t}$
$\ln(Distance_{I,J})$	-3.3968***
	(0.2493)
$\ln^2(Distance_{I,J})$	0.2301***
	(0.0229)
Within EA	-7.2402***
	(0.6348)
Neighboring EA	0.5792***
	(0.0553)
# Inventors (ln)	0.9208***
	(0.0223)
# Industry Workers (ln)	-0.1192
	(0.0929)
# Plants of 200+ Employees (ln)	-0.0345
	(0.0338)
Fixed-Effects	
Origin dummies	Yes
Destination dummies	Yes
Time×Tech	Yes
Fit statistics	
Observations	6,055,859
Adj-pseudo R^2	0.87615
Log-Likelihood	-899,039.62
BIC	1,857,578.26

Notes: Two-way clustered standard-errors in parenthesis (on I and J EAs). Level of statistical significance: ***, **, * means significance at the 1%, 5% and 10% level.

In a second step, we compute the numbers of collaborations between EA-Tech, using the predicted value from Equation (15). Such collaboration flows between EA-Tech are thus only correlated to variables that are controlled for in the second stage of the IV. The number of collaborations between each pair of EA-Tech is generated according to independent draws from a Poisson law, $\widetilde{Collab}_{I,J}^{f,t} \sim Poisson\left(\hat{\lambda}_{I,J}^{f,t}\right)$, where $\hat{\lambda}_{I,J}^{f,t}$ is the expected predictor of the estimation represented by Equation (15). In the third and last step, having positioned the same number of nodes in each EA-Tech as in the real network, and for each assigned collaboration between two EA-Tech, we then *pick at random* one inventor in each EA-Tech and create a link between them. Such random assignment affects slightly the degree distribution as connections are more asymmetrically distributed in the real network than in the generated counterpart (See Figure 6).



distribution.pdf

Figure 6: Degree distribution of the observed and generated networks (all inventor-years).