Reference classes: A tool for benchmarking universities' research

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Abstract

Based on new comparison principles that take into account both the volume of scientific production and its impact, this paper proposes a method for defining reference classes of universities. Several tools are developed in order to enable university managers to define the value system according to which their university shall be compared to others. We apply this methodology to French Universities and illustrate it using the reference classes of the best ranked universities according to several value systems.

Dominance relations citations impact reference classes

1 Introduction

Facing rising international competition for reputation, funds, scholars and students, the managers and the various stakeholders of universities increasingly need to define their strategic orientations, based on clear information on the positioning of their institutions in relation to the others. In this respect, benchmarking exercises, which enable a given institution to learn from peers, could be highly useful, if, of course, these peer institutions were selected on the basis of explicit and meaningful criteria, linked to the very strategic choices faced by the institution.

Thus we designed a model aimed at providing a benchmarking tool, enabling university managers to be involved in the definition of the value systems used to select the peer universities (or peer departments) with which their institution are to be compared. This set of peers constitute the *reference class* of the university. We build the reference classes by using *dominance relations* which constitute an explicit and natural tool for ordering pairs of universities. Here the dominance relations are based both on the articles published by the universities and their impact (with different measures of it).

The first (and recently rediscovered) synthetic measurement of both the quantity and impact of articles was proposed by [7]. It is computed as the average number of citations times the squared root of the total number of citations. Interest in this question has recently been spurred by Hirsch's introduction ([6]) of the so-called h-index, an index precisely designed to simultaneously account for both quality and quantity in a specific manner. Though this index has been highly discussed,¹ few papers have sought to explain the implicit value judgements of the h-index and of its variants. That very interesting line of inquiry, followed by [10] and [8], consists in picking the desired index and building an axiomatic which explicits its value judgements. To our knowledge no academic article yet has attempted to derive comparisons of scientific productions from explicit value judgements taking into consideration both their quantity and impact. This constitutes the basic rationale behind our dominance relations.

As a point of departure, we derive a quality measure associated to any impact level in any given discipline. Then, we define a function which attributes a value to any given level of quality of an article. We assume that the total value of institutions' research is the sum of the value of all its articles recorded. The dominance relations establish that the value of the production of a given institution is greater than the one of another institution for any value function within some well defined class of such functions. Classes of value

¹Many contributions have aimed to overcome such shortcomings (especially of the latter kind), the g-index ([4]), the tapered h-index ([1]), w-index ([10])...

function incorporate value judgements on the relative valuation (and its form) of quantity and quality. We propose three types of dominance relations. The strong dominance is based on the volume of articles only (no consideration of their impact). The (simple) dominance relation assumes that impact shall be taken into consideration positively without further specification. The weak dominance relation puts some emphasis on high impact scores, so that for instance an article weighing ten will be more prized than two articles weighting five.

We have shown elsewhere that dominance relations can be used for compiling university rankings ([2]). In this paper, we intend to show that it can be used to define sets of university peers, the so-called reference classes. For any given university, its reference class is the set of those universities that do not dominate it nor are dominated by it, the very universities along which to be benchmarked. The two developments, ranking and reference classes, can actually be seen as the two faces of the same coin, since they are two different ways of exploiting the same basic one-to-one dominance relation model. They can also be seen as complementary tools, to be used for analyzing the relative positioning of the universities.

Given the genericity of the approach, we are able to build customized reference classes, by selecting the parameters of the value system associated with the one-to-one dominance relations along which a given institution will be compared to the others. The method that we propose for ranking the universities can be seen as occurring in a *four-dimensional space* in which each dimension is customizable.

The first dimension is the type of dominance relation selected among the three exposed above. The second dimension concerns the impact of the articles. This dimension can be directly measured by counting the number of citations received by the articles. Though this measure is attractive, it is also interesting to appreciate the impact by using the journals' Impact Factors, since the latter accounts for the ability to be selected by and published in well established, influential journals. A third alternative measure is the Relative Impact Factor of the journals, which normalizes the Impact Factor at the specialty level. This measure is more comprehensive for all scientific fields, since it includes all the best journals in every specialty, whatever their relative impact. The third dimension is about the impact threshold the articles must reach in order to be considered. For instance, only taking into consideration the world's most visible articles would mean that the managers only wish to consider production at global level in their comparative analysis. The fourth dimension is about the disciplinary focus: reference classes can be built either for one given discipline or for all disciplines. The former is more designed for benchmarking departments whereas the latter is better suited for

comparing universities.

We believe that all these possible ways of building the reference classes make our tool easy to adapt in such a way as to take into account the managers' various points of view about what is the most for comparing their university to others. One specific manner to set these four dimensions is called here a *value system*.

Here, we illustrate the proposed model by applying it to French universities and one discipline: fundamental biology. All French universities are compared one-to-one using the proposed value systems. The data comprises detailed information on the publications of all French Universities as they appear in the ISI-WOS-OST database.² These data are highly reliable thanks to the use of an interactive data collection and cleaning process involving the universities themselves so as to take into consideration the variety of signing practices among the research staff of these institutions, and the "fuzzy boundaries" of French universities. The methodology is to be validated (and hopefully improved) by a selected set of university managers and experts. Then the results are designed to develop a tool which could be adapted at a larger international scale.

The article is organized as follows. The theoretical foundations of our approach are introduced in the second section. The third section presents the notion of reference classes and how the user can use this tool for different applications. In the fourth section we discuss and apply these concepts and techniques to a set of the largest French universities (disciplinary focus). The last section concludes.

2 Theoretical foundations

This section is devoted to presenting the theoretical foundations of the definition of reference classes. The reference classes are established on the basis of associated dominance relations which constitute the main theoretical foundations of our model. They rely on clear cut sets of conditions which may correspond to the specific goals of different "examiners" (whoever they may be, either a university stakeholder, the government, students...). Therefore, the theoretical developments below will also provide the foundations of the four dimensional-value systems associated to reference classes. In the next section, we shall explain more thoroughly how dominance relations are used as a basis for defining reference classes.

²These data are managed and enriched in house by the Observatoire des Sciences et Techniques through a very detailed and precise techniques that involves directly the institutions for the selection of the appropriate list of addresses mentioned in their publications.

2.1 Scientific production: measure

Consider set I of n research institutions i = 1, ..., n and let us denote a an item in A the set of all articles produced by these institutions. An impact measure $x_a \in \mathbb{R}^+$ is associated to each article a. The production of an institution i is now described by a $1 \times n_i$ vector $x_i := (x_1^i, x_2^i, ..., x_a^i, ..., x_{n_i}^i)$, with n_i the total number of items produced by institution i.

The structured set of actor *i*'s publications in domain $k \in K$, the set of all scientific disciplines, is given by: $\{f_i^k(x) | \forall x \in \mathbb{R}^+\}$. $f_i^k(x)$ is the publication performance of *i* with impact *x* in domain *k*. The question of how the impact can and should be measured will be discussed further in the next section. Now we take this as given and focus on the computation of performance. In this article we use the so-called *fractional count* measure. Let article *a*, referencing at least one address associated to institution *i*, bring a score of:

$$p_{i,a}^{k} = \frac{\# \{ \text{address of } i \text{ co-occurs in } a \}}{\# \{ j | j \text{ is a co-autor of } a \}} \times \frac{1 \{ k \in d \left(j(a) \right) \}}{\# d \left(j(a) \right)}, \tag{1}$$

with $j(a) \subset A$ denoting the subset of all papers published in the same journal as a and the term d(j) the set of disciplines to which journal j is to be associated. The expression $\#\{.\}$ denotes the cardinal of the set defined between brackets, the numerator of the first fraction on the right hand side of (1) indicates the number of times i is mentioned in the list of institutions mentioned by the authors of the article, and 1 $\{.\}$ is the indicator function which equals 1 if the condition in the brackets is true and zero otherwise. Thus, the first ratio of the right hand side of (1) indicates the weight of institution i among the various institutions mentioned by the authors of article a. The second ratio serves as a filter for selecting the articles related to discipline k, through the association of the journal in which it was published to one or several disciplines, and it helps give weight to discipline k when the journal is related to several disciplines. Then the performance of institution i in domain k, noted $f_i^k(x)$, given any level of impact x, is computed by:

$$f_i^k(x) := \sum_{a=1,\dots,n_i} 1\{x_a^i = x\} \times p_{i,a}^k$$

 $f_i^k(x)$ is non negative and becomes nil once x reaches a certain value, which varies across institutions and scientific disciplines.

2.2 Dominance relations

In this subsection, we first present the notion of valuation function, which is a necessary step before introducing the dominance relations. Then, since the mathematical processing and analysis of dominance relations differ depending on whether one considers comparisons within or between disciplines, we have split the presentation of dominance relations into two parts. The former are better suited for the comparison of university departments, while the latter may be used to compare universities.

2.2.1 Valuation functions

Let us now denote $v(\cdot) : S \to \mathbb{R}$ the valuation function which gives the "value" of any article given its quality, $S \subseteq \mathbb{R}^+$ is the set of all possible quality measures. The value of the whole production performance of institution *i* is given by:

$$V_i^k = \sum_{s \in S, s \le \bar{s}} v(s) f_i^k(s) \, ds, \tag{2}$$

with $\bar{s} = \min s > \max_{i \in I} \max_{j=1,...,n_i} s_j^i$, the lowest quality, which no article produced by the agents in set *I* could reach (it provides a strict upper bound to the quality of the articles produced in I.

The simplest way of dealing with quality in this context would be to assume that impact is the right measure of quality. However, we do not retain this assumption because impact varies dramatically from discipline to discipline, due to varying citation practices across disciplines. We therefore propose to measure articles' quality through their relative position in the distribution of articles (according to their impact) within their corresponding discipline: the quality of a given paper a in discipline k is s_a^k if it is equal to the maximum s such that its impact x_a is least as high as that of exactly $100 \cdot s$ percent of the articles published in discipline k. In other terms, its quality is equal to the probability that a randomly drawn article in discipline k has a lower (or equal) impact. Formally, let $\{\varphi^k(\cdot), \forall x \in \mathbb{R}^+\}$ be the density distribution in discipline kof all production according to an impact measure scaled by x and $\Phi^k(\cdot)$ the associated cumulative distribution. The quality of paper a is thus $s_a^k = \Pr(X^k \leq x_a) = \Phi^k(x_a)$.³ This will enable us to aggregate the articles produced in the different disciplines for any quality level.

2.2.2 Intra-discipline dominance relations

Three types of dominance relations are defined: strong dominance, dominance and weak dominance. Each dominance relation is associated with various assumptions on function v(s). Dominance relations are established beyond a certain threshold of quality, that is by only considering the best articles in the discipline. To do so, we first need to

³See Carayol and Lahatte (2011) for details.

define $\phi \in [0, 1]$ share of the highest quality papers in discipline k (in the world) to be considered.

Definition 1 The scientific production, in discipline k, of institution i (a) strongly dominates, (b) dominates or (c) weakly dominates at order $\phi \in [0,1]$ the one of institution j, noted (a) $i \blacktriangleright_k^{\phi} j$, (b) $i \rhd_k^{\phi} j$ or (c) $i \trianglerighteq_k^{\phi} j$, if $\sum_{s \in S, s \ge 1-\phi} v(s) f_i^k(s) \ge$ $\sum_{s \in S, s \ge 1-\phi} v(s) f_j^k(s)$ for any (a) positive function (b) positive and non-decreasing (c) positive, non-decreasing and weakly convex function $v(\cdot)$.

The three notions of dominance require the unanimity of judgments associated with value functions belonging to different classes of functions. The notion of strong dominance only requires that function v(s) be non-negative, i.e. that no article has a negative effect on the scientific performance of any institution. This very weak assumption implies that the impact of articles plays almost no role. The notion of dominance requires function v(s) to be non-negative and non-decreasing, i.e. that articles of a higher quality have a higher valuation (within a given domain). This assumption is also likely to be considered acceptable. The notion of weak dominance requires, in addition to the above mentioned properties, that function v(s) be also weakly convex. This assumption implies that the value function gives proportional or more than proportional weight to the best papers in terms of quality. If v(s) accounts for the contribution of the paper to the prestige of the parent institution, then the weak convexity assumption may be accepted as relevant by most universities' CEOs and their trustees since they usually want their institution to produce very high impact articles and grant little attention to lower impact works. Evaluation procedures which only consider high quality articles (such as the British Research Assessment) rest heavily on this (implicit) assumptions.

Carayol and Lahatte in [2] establish the necessary and sufficient conditions for each dominance relation to hold, relying only on information about publications and their impact when the impact threshold is zero. We here introduce a slightly extended proposition to the situation in which $s_k^{\phi} \geq 0$.

Proposition 1 The three following statements hold:

$$\begin{array}{l} i) \ i \blacktriangleright_{k}^{\phi} \ j \ iff \ \forall u \in [1 - \phi, 1] \,, \ \left[f_{i}^{k} \left(u \right) - f_{j}^{k} \left(u \right) \right] \ge 0; \\ ii) \ i \vartriangleright_{k}^{\phi} \ j \ iff \ \forall u \in [1 - \phi, 1] \,, \ \sum_{s \in S, s \ge u} \left[f_{i}^{k} \left(s \right) - f_{j}^{k} \left(s \right) \right] \ge 0; \\ iii) \ i \trianglerighteq_{k}^{\phi} \ j \ iff \ \forall u \in [1 - \phi, 1] \,, \ \sum_{s \in S, s \ge u} s \left[f_{i}^{k} \left(s \right) - f_{j}^{k} \left(s \right) \right] \ge 0. \end{array}$$

Proof. Straightforward extension of the proofs presented in [2].

These results are important because they make it possible to compute the dominance relations, without having to further specify the functional forms of the various dominance relations. These results also provide a different perspective on the meaning of the difference dominance relations. Strong dominance requires that the dominating institution perform better than the dominated one in all the quality segments. Therefore, strong dominance treats all quality levels in the same way, that is no specific premium is given to the highest quality articles. The dominance relation requires that the total publication performance of the dominating institution above any quality level to be higher than that of the dominated institution. Weak dominance gives an even greater role to quality in that it weighs articles according to their quality level. Therefore, though both dominance and weak dominance value quality, weak dominance gives a greater weight to the concentration of scientific production in high quality classes.

2.2.3 Interdisciplinary dominance

We here adopt an interdisciplinary approach which aggregates disciplinary production before dominance relations are established.

Definition 2 The scientific production of institution i (a) strongly dominates, (b) dominates or (c) weakly dominates, at interdisciplinary level, at order $\phi \in [0,1]$ the one of institution j, noted (a) $i \triangleright^{\phi} j$, (b) $i \triangleright^{\phi} j$ or (c) $i \trianglerighteq^{\phi} j$, if $\sum_{s \in S, s \ge 1-\phi} \sum_{k} v(s) f_{i}^{k}(s) \ge$ $\sum_{s \in S, s \ge 1-\phi} \sum_{k} v(s) f_{j}^{k}(s)$ for any (a) positive function (b) positive and non-decreasing (c) positive and non-decreasing and weakly convex function $v(\cdot)$.

Again, we have necessary and sufficient conditions for each dominance relation to hold relying only of publication and impact information (now in all disciplines).

Proposition 2 The following three statements hold:
i)
$$i \triangleright^{\phi} j$$
 iff $\forall u \in [1 - \phi, 1], \sum_{k} \left(f_{i}^{k}(u) - f_{j}^{k}(u) \right) \geq 0;$
ii) $i \triangleright^{\phi} j$ iff $\forall u \in [1 - \phi, 1], \sum_{s \in S, s \geq u} \sum_{k} \left(f_{i}^{k}(s) - f_{j}^{k}(s) \right) \geq 0;$
iii) $i \trianglerighteq^{\phi} j$ iff $\forall u \in [1 - \phi, 1], \sum_{s \in S, s \geq u} \sum_{k} s \left(f_{i}^{k}(s) - f_{j}^{k}(s) \right) \geq 0.$

Proof. Trivial extension of the proof of Proposition 1.

3 Reference classes

We now turn to the definition and implementation of reference classes. For this purpose, we first introduce the notion of reference classes and present their main properties, before parameterizing the tool according to the goals selected.

3.1 Reference classes

A strict natural ordering (dominance relation) cannot always be established between two institutions. This occurs when at least two valuation functions of the associated class lead to opposed orderings of the scientific productions of the two compared institutions. According to the explicit value system previously chosen, the publication profiles of the two institutions can not be naturally ordered. What does this mean in more concrete terms? That the two universities under examination are not different enough to be "naturally" ordered: for example, when one university dominates the other in one set of impact levels while the other's performance is higher in another set of impact levels, in such a way that unanimity cannot be obtained within an assembly of examiners (though they share a common value system) who may then conclude that the two universities are "peers" to some extent : they belong to the same reference class.

The notion of reference class builds precisely on this idea. The reference class of institution i, associated with a dominance relation \succ , is the set of institutions noted $c_i^{\succ} \subseteq I$. Dominance relation \succ could be any one of the dominance relations examined above. The construction of reference classes can thus be modulated by choosing the associated underlying dominance relation. The formal definition follows.

Definition 3 $\forall k \in I, \ k \in c_i^{\succ} \subseteq I \ if \ i \neq k \ and \ k \neq i \ or \ if \ i \succ k \ and \ k \succ i$

Notice that then all institutions belong to their own reference class and that the relation is reciprocal in the sense that $k \in c_i^{\succ}$ iff $i \in c_k^{\succ}$.

One interesting property of the reference classes (synthesized in Theorem 1 below) is that the weaker the associated dominance relation, the more the reference classes are reduced to a smaller set of institutions. Therefore it is possible, to some extent, to adjust the size of the reference class by strengthening or weakening the associated dominance relation. For instance, the theorem implies that, above any given impact threshold, the reference class of any university based upon the dominance relation is included in the reference class of any university based upon the strong dominance relation (for that threshold). Furthermore, for any given dominance relation, the reference class of a university based on an impact threshold, is included in its reference class for any lower impact threshold. These statements apply to the intra-discipline comparisons as well as to the cross-disciplinary and interdisciplinary comparisons.

Reference class properties build upon dominance relations properties.

Theorem 1
$$\forall \phi, \phi' \in [0, 1]$$
 such that $\phi \ge \phi', \forall k \in K$ and $\forall m < |K - 1|, \forall i \in I :$
1. $c_i^{\stackrel{\phi}{=}k} \subseteq c_i^{\stackrel{\phi}{=}k} \subseteq c_i^{\stackrel{\phi}{=}k};$

$$\begin{aligned} & 2. \ c_i^{\blacktriangleright_k^{\phi'}} \subseteq c_i^{\flat_k^{\phi}}, \ c_i^{\triangleright_k^{\phi'}} \subseteq c_i^{\triangleright_k^{\phi}} \ and \ c_i^{\succeq_k^{\phi'}} \subseteq c_i^{\succeq_k^{\phi}}; \\ & 3. \ c_i^{\succeq_{\phi}^{\phi}} \subseteq c_i^{\triangleright_{\phi}^{\phi}} \subseteq c_i^{\blacktriangleright_{\phi}^{\phi}}; \\ & 4. \ c_i^{\flat_{k_1,\dots,k_m}^{\phi'}} \subseteq c_i^{\flat_{\phi}^{\phi'}} \subseteq c_i^{\triangleright_{k_1,\dots,k_m}^{\phi}} \ c_i^{\succeq_{k_1,\dots,k_m}^{\phi'}} \subseteq c_i^{\succeq_{k_1,\dots,k_m}^{\phi}}, \\ & 5. \ c_i^{\flat_{k_1,\dots,k_m}^{\phi}} \subseteq c_i^{\flat_{k_1,\dots,k_m}^{\phi}}, c_i^{\flat_{k_1,\dots,k_m}^{\phi}}, c_i^{\succeq_{k_1,\dots,k_m}^{\phi}} \subseteq c_i^{\succeq_{k_1,\dots,k_m}^{\phi}}. \end{aligned}$$

Proof. See Appendix A.

3.2 Dominance networks

We here define a complementary tool, namely networks of dominance relations between institutions. Let us consider \succ , which could be any one of the dominance relations examined above $(\blacktriangleright_k^{\phi}, \triangleright_k^{\phi} \text{ or } \succeq_k^{\phi}, \text{ with } \forall \phi \in]0, 1]$). Let us build the (directed) dominance network \vec{g}_{\succ} associated to dominance relation \succ and the institutions set I by establishing a directed link from institution $i \in I$ to institution $j \in I$ ($j \neq i$) if i dominates jaccording to \succ . That is formally: $\forall i, j \in I, ij \in \vec{g}_{\succ}$ if $i \succ j$.

In this network, transitive dominance triplets (without ex-aequos) are uninformative since we know that the transitivity property holds. Therefore, for picturing purposes, it is convenient to define the adjusted dominance network $\vec{g'}_{\succ}$, which is derived from \vec{g}_{\succ} , by eliminating such triplets. Formally to build $\vec{g'}_{\succ}$, we begin by assigning a link from *i* to *j* in such a network if $ij \in \vec{g}_{\succ}$. But some links are deleted according to the following rule: $\forall i, j, h \in I$, if $ij, jh \in \vec{g}_{\succ}$ and $hj \notin \vec{g'}_{\succ}$ then $ih \notin \vec{g'}_{\succ}$. The condition whereby *h* must not dominate *j* enables us to avoid eliminating the link from *i* to *h* when *j* and *h* dominate each other (which basically means they have identical productions).

One can see the reference classes as a tool for the local exploration of dominance networks. The correspondence between the two notions is straight forward: if $i \in c_j$ (thus $j \in c_i$) then there is no path from i to j or from j to i on $\vec{g'}_{\succ}$.

3.3 Defining reference classes

The theory introduced above allows for four choices in building reference classes through the definition of a four dimensional-value systems. It consists, first of all, selecting the type of dominance relation; Second of all it consists in in selecting a measure for the impact of the articles - either the number of citations, or the impact factor; Thirdly, it implies the definition of a threshold on impact which will be used to select the articles to be considered. Lastly, it implies choosing between selecting one specific discipline or performing an interdisciplinary comparison.

3.3.1 Selecting the type of dominance relation

Three types of dominance can be selected. The strong dominance notion refers more to a volume of publications; this is simply because an institution can not strongly dominate an other institution if the latter has published more papers than the former for at least one impact level. Therefore, the reference classes of a given university i based on strong dominance group together the universities that are of the same scale as i in terms of scientific production, in the sense that none outperforms i, nor vice versa.

The (simple) dominance relation implies that any paper produced by a university always makes it possible to compensate for a lower impact article produced by the university it is compared to. This compensation regime (not allowed in the strong dominance) can help establish more dominance relations than in the strong dominance and therefore tends to take some institutions out of the reference classes. Basically, this leads to a configuration in which one institution can not be dominated by any other if it has more publications over any given level of impact. This gives small but excellent institutions the opportunity to position themselves in the same category as the larger institutions because they may have many articles in the very high impact segments. However this compensation regime does not allow more than this and therefore, large institutions, which produce more articles, are not easily dominated by smaller ones. This also gives large institutions a chance to have as peers smaller universities, which produce more high impact articles.

The weak dominance relations provide a much clearer focus on the production of higher impact articles. This is simply because an institution cannot be dominated by an other if it performs better in the highest impact segment. Therefore weak dominance is not directly a size-dependent indicator. It gives a crucial role to the highest impact articles when comparing two institutions. Therefore, a small but highly visible school or university may find it more relevant to build its reference class using the weak dominance because it leads to define peers that, though they may produce much more articles globally, are in fact of similar importance in their higher segments of impact.

3.3.2 Selecting a proxy for measuring scientific impact

Three proxies are proposed here. First, scientific impact can be measured by counting, for each article published, the number of citations received in a given time window. The idea that the academic credit can be approximated by citations dates back to the pioneering work of sociologist R. K. Merton (Cf. his collected articles in [9]) and was expanded by scientometricians such as [3] and [5]. Such a measure is built from the

citations made directly to the articles and is thus very attractive . It is however a very noisy measure: scholars often complain that their best papers are not the most cited . Papers happen to be cited for accidental reasons rather than for their real contribution.

Therefore, one may alternatively consider the journals Impact Factor as an appropriate (though more indirect) measure for scientific impact. It is computed as the average number of citations received by articles published in the journal. Such a measure of impact accounts more for the ability to publish in well established journals, that are read and cited by a large audience. Clearly, the universities that perform well when impact is computed this way have a high academic reputation in the largest communities of the discipline, as shown by their ability to publish in those highly visible journals. This measure (as well as the one previously mentioned one) has the drawback of favouring the most visible specialties or communities (sub-disciplines) in a discipline. The last measure corrects for such a potential bias.

The last measure of impact is the Relative Impact Factor, which is a measure of a journal's impact factor compared to the average impact factor of the journals of the same specialty. Such a measure is particularly useful when considering that the fields of specialization are given and that the best researchers can do is publish their articles in the best journals of their fields. Such a measure also controls for the various citation practices in the various specialties of the same discipline (e.g. applied and fundamental mathematics).

3.3.3 Selecting the articles to take into consideration

The third dimension is the impact threshold, which basically sets the level of impact necessary in order to be taken into account in the analysis. In principle any percentage may be chosen. To illustrate and clarify the idea, we will consider two different cases: $\phi = 1$ and 0.1. In a national comparison context, one may considers all articles ($\phi = 1$). International comparisons focusing on the top universities may limit relation considers all articles among the 10% most cited in the field ($\phi = 0.1$).

3.3.4 Selecting a disciplinary focus

Our model makes it possible to restrict the analysis to any given discipline. In this case, the comparisons are adapted to departments. It must be noted however that the boundaries between disciplines are determined by the allocation of journals to disciplines. We do not compare departments directly (a paper published in an applied mathematics journal and authored by a member of the biology department in question will be associated to the university's performance in mathematics, not biology). Interdisciplinary comparisons are more useful for comparing universities. Such an approach would then be preferable when one aims to find the peers of university i in terms of overall scientific impact, irrespective of the fact that these institutions may actually be good in very different domains to i, one may even be specialized in some disciplines while i could remain a generalist university.

4 Reference classes among French universities

We now apply our model to French universities. The data and data collection are described first. Next, the concept of reference classes is illustrated using the set of the highest ranking universities in the domain of fundamental biology.

4.1 The Data

The data come from the IPERU (Production indicators for French university research institutions) operated by the OST (French Science and Technology Observatory). Due to the complexity of the French research system, there is a great variety of patterns of referencing the employing institution which in turn makes it difficult to record the publication output of universities. To overcome this difficulty, the program relies upon the validation, by the universities' research support services, of the correct list of signing patterns their scholars and researchers use. The initial list of candidate institution names was established by the OST. It comprised all the institution names observed in the publications associated to the geographic zone in which each university is located (postal codes provided by the institutions). This work is carried out annually and concerns the French universities and Grandes Ecoles associated to the French ministry of research and higher Education and which are not fully specialized in the social sciences and humanities.⁴ The data on publications and citations used in this study relate to the identification which occurred in year 2008. It covers 129 actors.

The raw publication data come from the SCI-expanded database (Thomson-Reuters) which constitutes a reference product in scientometrics studies. It contains the standard scientific information on all articles published in a maintained list of about 6,500 journals. These journals are selected on the basis of their impact factor, their regularity and their compliance with some editorial criteria (such as peer review, rules for referencing

⁴Thus, the study does not concern the schools that are associated to other ministries (Defense, Industry and Agriculture mainly).

authors and cited articles). The journals are associated to scientific specialties (potentially to several ones) which can be aggregated in nine large disciplines. The first eight, presented in Table 1, do correspond to scientific disciplines in the classic sense of the term; whereas the ninth, labeled Multidisciplinary Sciences, corresponds to journals in which articles from different disciplines can be published. Thus in all the disciplinary based dominance studies, and so as to control for the bias resulting from not taking into account a significant part of the best articles in several disciplines, the articles published in PNAS (Proceedings of the National Academy of Sciences of the USA), Science and Nature have been reallocated to their parent discipline through a procedure of lexicographic recognition.

The impact is measured in different ways. Firstly, it is measured by the citations received by the considered articles. Since the cleaned publication data are available from 2002 only, and the data on citations only covers the period up to 2007, a three-year publication period (2003, 2004 and 2005) and a three-year citations window (2003-2005 for year 2003, 2004-2006 for year 2004 and 2005-2007 for year 2005) were selected. Secondly, it is also measured by the impact factor of the journals in which the considered articles were published. The impact factor accounts for the average number of citations received by articles in the journal. Thus, this measure accounts for the ability to publish in a good journal rather than the direct impact. It is also less noisy than citations. Lastly, we also consider the relative impact factor; this means that the impact factor of the journal is now bench-marked against the average impact factor of the journal' specialty. This measure makes it possible to correct for the different citation practices across subject categories within the same discipline (e.g. between applied and fundamental mathematics).

Table 3 presents the basic characteristics of the reference classes obtained for two large disciplines (fundamental biology and physics) and when an interdisciplinary approach is adopted. In each case, Table (3) reports the average, median and maximum sizes of the reference classes according to three impact measures used and three dominance types defined. The values of the means provided in the table are clearly consistent with the properties of the reference classes that imply that the weaker the associated dominance relation, the smaller the reference classes. The average and median sizes are small in comparison with the maximum size.

4.2 Reference classes: an application to fundamental biology

Our comments are restricted to the field of fundamental biology though we also present the reference classes in the fields of medicine (Table 9 and 10), physics (Table 11 and 12) and those built on an interdisciplinary basis (Table 13 and 14). This explanatory exercise limits itself to the exposure and discussion of the reference classes of the top seven universities. We do not consider strong dominance relations, which are based on weak assumptions on the implicit value function: basically the strong dominance relation does not use the information on the impact levels for comparing and thus provides information that is mostly related to the size of the university. For different reasons, we do not use the information on the impact levels of the articles and consequently the results are highly dependent on the size of the universities. Thus only dominance relations will be considered in the example. Two levels of ϕ are also used, 1 and .1, which would be the most suitable levels for comparisons in a national and an international context respectively. The three proxies of impact introduced are used.

Three basic types of information are manipulated so as to fully explain what happens in this example:

i) The scores of the institutions across the various levels of impact according to the three proxies used for measuring impact. This helps to better understand where the dominance (or non-dominance) comes from in the distribution. In practice we present the data corresponding to the decumulative functions, that is $h_i^k(x) = \int_x^{\infty} f_i^k(s) ds$ so as to assess dominance for various levels of ϕ . The corresponding data are presented in Tables 4, 5 and 6.⁵

ii) The networks of dominance relations provide a larger picture than just reference classes (Figure 1 to 6). They are especially helpful to understand what happens when one changes one dimension of the underlying value system.

iii) The reference classes (Table 7 and 8).

Let us first consider the case corresponding to $\phi = 1$ and citations as a measure of impact. We find that Paris 6 and Paris 11 are at the top of the dominance hierarchy, essentially for size-related reasons (with Paris 6 dominating Paris 11). Strasbourg 1 and Aix Marseille 2 come in a second set. They are in their respective reference classes because Strasbourg 1 performs better in the most cited segments while Aix Marseille 2 on the whole, produces a few more articles. A third subset comprises Paris 5, Lyon 1, and Paris 7. Lyon 1 and Paris 7 are immune to the domination of Paris 5 because they perform better in some intermediary segments of impact: Paris 5 performs better both in the highest segments of impact and in the lowest ones. Paris 5 also performs better than Aix-Marseille 2 in the highest segments of impact. Montpellier 2 constitutes an interesting case since its remarkable performance in the highest segments of impact (essentially its papers among the 5% most cited in the field) gives it immunity to the

⁵The function $g_i^k(x) = \int_x^\infty s f_i^k(s) \, ds$ would be used to assess weak dominance relations.

dominance of several larger universities (in this domain) such as Paris 5, Lyon 1, Paris 7 and even Aix Marseille 2 which are thus in its reference class.

When the impact factor of the journals is used as a proxy for impact, Paris 11 and Strasbourg 1 become immune to the dominance of Paris 6, both because they manage to perform better among the 5% most cited papers in the field (Strasbourg 1 performing even better than Paris 11 in this highest segment of impact). The teams in the departments of fundamental biology of these two universities seem to hold very strong positions and a high reputation for quality which enables them to publish papers in the highest circulation journals in their field. Aix-Marseille 2 now dominates Paris 5 and Montpellier, which hold relatively weaker positions in the most prestigious journals than they do from the perspective of the direct citation their articles receive.

When the impact is measured through the relative impact factor, we compare the universities' ability to publish in the best journals in the fields they specialize in. The relative impact measure makes it possible to appreciate the academic reputation within specialized communities whereas the direct impact factor accounts for the reputation within whole scientific disciplines. Then again Paris 6 dominates both Paris 11 and Strasbourg 1, the latter being immune to Paris 11's dominance thanks to greater score in the highest segments. Montpellier 2 is now produced a greater number of articles in the highest impact class (in the best 5% in the discipline)

When the comparisons only include the top ten percent most-cited papers in their field, we obtain a picture that only corresponds to the top of their respective publication distributions. According to Theorem 1, the reference classes at $\phi = .1$ are necessarily subsets of their corresponding ones (everything else remaining equal) at $\phi = 1$. This is because a dominance at $\phi = 1$ implies a dominance at $\phi = .1$ as one can see in the dominance networks (see Figures). This of course tends to favor the universities whose scientific production is concentrated in the top 10% of the discipline. For instance, when one takes the citations received as the impact measure, Strasbourg 1 dominates Aix-Marseille 2 and Paris 5 dominates Paris 7. When the impact factor of the journals is used, Strasbourg 1 dominates Paris 11, but none of them dominates nor are dominated by Paris 6 (they are in Paris 6's reference class). This clearly synthetically illustrates the high performance level achieved by Strasbourg 1 in the top journals of the discipline. Montpellier 2 also dominates Paris 5, Paris 7, Lyon 1 and Grenoble 1, though it has a smaller overall scientific production than these universities (and is surely of a smaller scale).

5 Conclusion

This article introduces a methodology for building reference classes on the basis of theoretically founded notions of dominance relations. The reference class of any university icomprises institutions that neither dominate nor are dominated by this university. This so-defined set of peer universities turns out to be a flexible benchmarking tool, its composition depending on several criteria such as the type of dominance relation, the type of impact, the impact threshold and, the discipline considered. We have illustrated how this methodology can be applied to a subset of all French universities using bibliometric indicators. We wish to make this tool available to the whole academic community.

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Appendix A. Properties of reference classes: proofs of Theorem 1

The properties of reference classes essentially build upon the properties of dominance relations. To show this we first need to introduce a useful definition that establishes that a dominance relation is stronger than another if a dominance relation of the former type between two institutions implies a dominance of the latter over the former, for any pair of institutions.

Definition 4 A dominance relation \succ is stronger than dominance relation \succ' , noted $\succ \gg \succ'$, if, $\forall i, j, i \succ j$ implies $i \succ' j$.

We can now introduces some causal relations between the dominance relations.

Lemma 2
$$\forall \phi, \phi' \in [0,1]$$
 such that $\phi \ge \phi', \forall k \in K$ and $\forall m < |K-1|$:
1. $\blacktriangleright_k^{\phi} \gg \triangleright_k^{\phi} : \stackrel{\phi}{\gg} \stackrel{\phi'}{\gg} \stackrel{\phi'}{\approx} and \stackrel{\phi}{\geq} \stackrel{\phi'}{\approx} \stackrel{\phi'}{\approx} \stackrel{\phi'}{\approx} \stackrel{\phi'}{\approx} and \stackrel{\phi}{\geq} \stackrel{\phi'}{\approx} \stackrel{\phi'}{\approx$



We now need to establish a correspondance between the relation between dominance relations and the relation between reference classes. This is the purpose of the following lemma.

Lemma 3 If $\succ \gg \succ'$ then $c_i^{\succ'} \subseteq c_i^{\succ}, \forall i \in I$.

Proof. Assume $\succ \gg \succ'$. Then, $\forall i, k \in I, i \succ j$ implies $i \succ' j$ which is equivalent to $i \not\prec' j$ then $i \not\prec j$. Now assume that $j \in c_i^{\succ'}$, which requires that either a) $i \not\prec' j$ and $j \not\prec' i$ or b) $i \succ' j$ and $j \succ' i$. If a) holds, then the definition of the " \gg " relation leads to $i \not\prec' j$ and $j \not\prec' i$, which in turn implies that $j \in c_i^{\succ}$. Now let us consider that b) holds. When $i \succ' j$, it is impossible that both $j \succ i$ and $i \not\neq j$ which simply means that it is impossible that $j \notin c_i^{\succ}$. Similarly, when $j \succ' i$, it is impossible that both $i \succ j$ and $j \not\neq i$,

that is, it is impossible that $j \notin c_i^{\succ}$. Therefore, necessarily $j \in c_i^{\succ}$ (and $i \in c_j^{\succ}$). So, it is possible to conclude that when $j \in c_i^{\succ'}$ then $j \in c_i^{\succ}$, which implies that $c_i^{\succ'} \subseteq c_i^{\succ}$. \Box

By application of the above Lemma, the five properties of reference classes in Theorem 1 now directly derive from the five properties of dominance relation in Lemma 1. QED

Appendix B. Tables and Figures

Univ	University full name
AM1	Aix marseille 1
AM2	Aix marseille 2
B1	Bordeaux 1
B2	Bordeaux 2
С	Caen
ENS	Ecole Nationale Supérieure de Paris
ENSI	Ecole Normale Supérieure d'Ingénieurs de Caen
ESPCI	Ecole Supérieure de Physique et de Chimie Industrielles de la Ville de Paris
G1	Grenoble 1
INPG	Institut National Polytechnique de Grenoble
Li2	Lille 2
L1	Lyon 1
M2	Montpellier 2
Na1	Nancy 1
P5	Paris 5
P6	Paris 6
$\mathbf{P7}$	Paris 7
P11	Paris 11
P12	Paris 12
R1	Rennes 1
S1	Strasbourg 1
T3	Toulouse 3

Table 1: List of the abbreviations of the institutions.

Table 2: The domains.

- k Domain
- 1 Fundamental biology
- 2 Medicine
- 3 Applied biology/ecology
- 4 Chemistry
- 5 Physics
- 6 Science of the universe
- 7 Engineering sciences
- 8 Mathematics

			Mean	16.59	12.63	12.58				Mean	16.58	10.50	10.44				Mean	15.97	9.17	8.97
			Max	73.00	69.00	69.00				Max	75.00	62.00	62.00				Max	32.00	21.00	21.00
	$\phi = .1$	ournal IF	Median	5.00	5.00	5.00		$\phi = .1$	ournal IF	Median	12.00	8.00	8.00		$\phi = .1$	ournal IF	Median	10.00	7.00	7.00
		Ū	Mean	8.70	6.94	6.92				Mean	15.88	10.81	10.69			[,	Mean	11.27	7.31	7.17
			Max	77.00	69.00	68.00				Max	52.00	47.00	47.00				Max	34.00	21.00	20.00
		Citations	Median	8.00	6.00	6.00			Citations	Median	11.00	8.00	8.00			Citations	Median	15.00	7.50	7.00
3io		U	Mean	12.20	9.41	9.34	cs			Mean	13.41	9.89	9.86	plines		0	Mean	14.36	8.13	7.91
Fund I			Max	113.00	86.00	81.00	Physic			Max	104.00	69.00	64.00	All Disci _l			Max	84.00	35.00	32.00
		Rel JIF	Median	60.00	29.00	28.00			Rel JIF	Median	72.00	26.50	24.00	ł		Rel JIF	Median	46.00	16.00	14.00
			Mean	54.11	32.30	30.22				Mean	62.13	26.17	23.92				Mean	44.63	15.83	13.70
			Max	112.00	87.00	83.00				Max	113.00	67.00	63.00				Max	93.00	46.00	33.00
	$\phi = 1$	Journal IF	Median	63.50	33.00	31.50		$\phi = 1$	Journal IF	Median	77.00	31.50	27.50		$\phi = 1$	Journal IF	Median	50.00	19.00	16.00
		`	Mean	57.81	34.47	32.00				Mean	67.92	30.34	27.77			`	Mean	49.13	18.91	16.03
			Max	92.00	75.00	75.00				Max	87.00	62.00	59.00				Max	95.00	48.00	38.00
		Citations	Median	52.00	28.50	26.00			Citations	Median	52.00	27.50	24.00			Citations	Median	61.00	17.00	13.00
		•	Mean	47.81	32.05	29.63				Mean	46.05	27.75	25.44			•	Mean	56.14	15.88	12.98
					\triangle	ΔI						\triangle	ΔI						\triangle	Δ I

Table 3: Statistics on the reference classes of all French universities

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World share	AM2	G1	L1	M2	P11	P5	P6	$\mathbf{P7}$	S1	T3
0.06	38.21	23.26	28.41	41.52	67.60	40.33	71.40	36.89	58.07	25.00
0.11	81.14	60.18	58.55	73.30	117.83	75.86	139.85	70.61	114.04	50.85
0.16	124.14	93.03	89.82	106.44	178.13	116.49	216.69	114.88	167.91	80.32
0.20	162.89	120.77	125.76	135.44	231.15	152.57	273.77	149.86	204.99	101.33
0.27	207.09	167.77	169.20	176.79	306.53	212.49	360.75	201.67	262.06	141.52
0.31	242.84	196.53	206.83	199.97	360.16	235.46	411.77	241.14	295.73	162.70
0.35	289.54	224.74	244.91	223.07	404.02	271.43	473.02	276.85	340.23	188.00
0.41	338.16	263.16	289.50	255.94	466.28	318.68	535.52	320.97	377.50	220.20
0.43	356.91	279.41	308.35	264.86	487.26	345.77	566.78	338.96	390.11	228.03
0.48	401.47	303.41	353.63	285.90	526.13	380.66	632.02	379.63	431.54	260.97
0.56	463.77	344.70	419.90	326.88	610.02	442.17	743.27	441.69	481.64	315.15
0.56	463.77	344.70	419.90	326.88	610.02	442.17	743.27	441.69	481.64	315.15
0.65	534.57	403.19	494.28	373.40	698.32	516.78	840.86	509.28	541.54	359.17
0.65	534.57	403.19	494.28	373.40	698.32	516.78	840.86	509.28	541.54	359.17
0.76	613.54	463.55	580.29	424.64	806.20	585.60	959.57	575.86	618.78	402.41
0.76	613.54	463.55	580.29	424.64	806.20	585.60	959.57	575.86	618.78	402.41
0.76	613.54	463.55	580.29	424.64	806.20	585.60	959.57	575.86	618.78	402.41
0.88	704.58	521.55	672.84	483.82	906.81	667.49	1076.95	643.91	686.58	451.71
0.88	704.58	521.55	672.84	483.82	906.81	667.49	1076.95	643.91	686.58	451.71
1.00	786.08	577.16	749.80	536.68	997.46	756.68	1206.97	714.04	756.94	503.41

Table 4: The (decumulative) scientific production of the highest ranking French universities in fundamental biology (among various percentages of the world's most cited articles)

World share	AM2	G1	L1	M2	P11	P5	P6	P7	S1	T3
0.06	51.23	31.41	25.96	43.96	73.45	32.99	71.97	26.92	83.13	24.54
0.11	104.47	57.42	74.01	95.03	129.16	87.27	183.94	80.82	167.27	58.75
0.18	178.98	133.79	108.95	141.84	241.63	150.50	276.13	162.10	243.07	125.98
0.23	217.46	170.17	168.97	178.98	320.69	203.98	355.39	225.83	301.80	158.12
0.28	271.82	203.07	211.31	216.00	405.47	247.47	446.04	287.42	329.62	187.89
0.34	316.58	230.49	263.01	254.97	473.74	291.75	526.44	335.17	380.94	209.01
0.39	364.75	270.47	318.62	282.94	521.41	346.18	604.17	376.47	432.95	232.20
0.44	420.45	293.68	360.99	306.94	585.53	384.66	665.48	409.14	466.28	254.20
0.49	465.66	320.10	404.24	328.65	638.76	421.92	715.17	448.97	504.01	281.74
0.53	504.23	349.08	435.74	358.80	691.66	458.82	773.58	482.03	541.29	307.88
0.58	540.44	375.05	468.66	380.10	729.43	491.03	819.09	501.22	566.83	336.97
0.62	556.66	388.97	498.08	403.88	759.39	518.66	872.51	529.58	590.48	361.19
0.67	594.23	412.56	551.56	424.02	799.73	543.21	925.91	559.22	622.26	376.74
0.72	631.02	453.97	590.07	440.84	836.07	576.92	982.36	592.68	640.86	398.43
0.77	671.12	475.25	624.07	457.90	874.68	616.17	1026.19	627.39	660.38	420.59
0.81	691.85	501.52	648.76	471.39	906.72	654.52	1060.92	650.64	690.48	438.27
0.86	708.97	525.08	672.47	485.55	938.99	674.69	1092.55	666.31	712.29	453.41
0.90	729.87	536.85	706.79	498.70	957.43	699.19	1128.54	681.96	727.23	476.50
0.95	761.71	552.23	725.19	507.37	972.05	710.84	1159.78	694.20	736.01	490.32
1.00	786.08	577.16	749.80	536.68	997.46	756.68	1206.97	714.04	756.94	503.41

Table 5: The (decumulative) scientific production of the best ranked French universities in fundamental biology (for various percentages of the articles with the world's highest direct impact factors.)

World share	AM2	G1	L1	M2	P11	P5	P6	P7	S1	T3
0.05	41.84	25.96	28.77	37.03	48.28	37.91	63.84	24.18	61.64	19.33
0.11	87.35	52.55	66.29	65.75	107.86	89.69	156.20	66.86	125.25	48.24
0.16	131.31	81.08	116.04	97.35	162.04	128.23	237.31	109.21	176.04	80.08
0.21	180.14	105.62	160.03	127.09	235.84	176.25	297.69	175.16	217.48	107.77
0.27	227.74	151.58	202.32	172.62	311.85	226.73	369.02	234.33	264.28	146.15
0.33	295.97	196.19	252.17	209.85	397.94	273.66	457.00	289.66	325.70	194.63
0.38	361.02	226.68	299.91	237.90	471.57	319.92	532.34	336.98	381.75	226.65
0.43	398.38	247.23	342.72	269.21	532.32	352.55	597.45	365.94	410.84	247.33
0.48	436.39	267.46	376.83	301.97	564.80	380.81	666.53	393.88	451.03	263.87
0.53	464.90	305.81	410.60	330.98	613.24	423.12	722.51	426.17	487.70	284.83
0.58	505.83	344.14	449.20	353.18	668.48	463.08	787.06	471.51	534.95	309.89
0.63	545.44	379.70	485.80	375.06	715.05	493.19	827.08	490.32	556.15	328.29
0.67	577.79	405.43	515.48	392.71	753.31	515.20	873.84	522.66	581.95	364.47
0.72	618.35	423.34	554.35	418.45	798.38	545.90	926.11	551.92	611.66	388.60
0.77	649.85	457.62	593.71	444.49	838.87	584.55	984.64	583.48	641.82	416.68
0.81	690.49	484.67	632.61	459.16	874.50	622.87	1037.78	620.68	674.97	435.23
0.86	714.02	506.03	670.70	481.93	908.31	666.13	1085.62	659.34	697.35	452.55
0.90	740.88	531.54	699.03	496.50	948.58	691.80	1121.95	675.88	720.32	469.72
0.95	763.84	550.91	726.96	511.09	971.97	712.71	1159.25	693.26	735.84	491.08
1.00	786.08	577.16	749.80	536.68	997.46	756.68	1206.97	714.04	756.94	503.41

Table 6: The (decumulative) scientific production of the best ranked French universities in fundamental biology (for various percentages of the articles with the world's highest relative impact factors.

	Citations	Journal IF	Rel JIF
AM2	M2, P5, S1	P7, S1	P5, P7, S1
G1	L1, M2, T3	L1, M2, P7, T3	M2, P7, T3
L1	G1, M2, P5, P7	G1, M2, P5, P7	M2, P5, P7
M2	AM2, G1, L1, P5, P7	G1, L1, P5, P7	G1, L1, P7
P11	-	P6, S1	$\mathbf{S1}$
P5	AM2, L1, M2, P7	L1, M2, P7	AM2, L1, P7
P6	-	P11, S1	-
$\mathbf{P7}$	L1, M2, P5	AM2, G1, L1, M2, P5	AM2, G1, L1, M2, P5
S1	AM2	AM2, P6, P11	AM2, P11
T3	G1	G1, L1	G1,M1,T3

Table 7: Reference classes of the best ranked French universities in fundamental biology, with dominance relation and $\phi = 1$.

Table 8: Reference classes of the best ranked French universities in fundamental biology, with dominance relation and $\phi = .1$.

	Citations	Journal IF	Rel JIF
AM2	M2, P5	-	P5
G1	L1, T3	L1,P7,T3	$\mathbf{P7}$
L1	G1	G1	M2, P7
M2	AM2, P5	-	L1, P7
P11	-	P6	-
P5	AM2, M2	-	AM2
P6	-	P11, S1	-
$\mathbf{P7}$	-	G1	G1, L1, M2
$\mathbf{S1}$	-	P6	-
T3	G1	G1	M1

	Citations	Journal IF	Rel JIF
AM2	-	-	Li2, P12
B2	Li2, T3	P12, S1, T3	Li2, P12, S1, T3
L1	P11	P11	P11
Li2	B2	-	AM2, B2, P12, S1
P11	L1	L1	L1
P5	-	-	-
P6	-	P7	-
$\mathbf{P7}$	-	P6	-
P12	-	B2,S1,T3	AM2, B2, Li2, S1, T3
T3	B2	B2,P12,S1	B2, P12, S1

Table 9: Reference classes of the best ranked French universities in medicine, with dominance relation and $\phi = 1$.

Table 10: Reference classes of the best ranked French universities in medicine, with dominance relation and $\phi = .1$.

	Citations	Journal IF	Rel JIF
AM2	-	-	Li2, P12
B2	Li2	P12, T3	Li2, P12, S1
L1	-	-	-
Li2	B2	-	AM2, B2, P12, S1
P11	-	-	-
P5	-	-	-
P6	-	P7	-
$\mathbf{P7}$	-	P6	-
P12	-	B2, S1, T3	AM2, B2, Li2
S1		P12, T3	B2, Li2
T3	-	B2, P12, S1	

	Citations	Journal IF	Rel JIF
AM1		B1, INPG, L1, M1, S1	B1, L1, S1, T1
B1	C, ENSI, L1, M2, S1	AM1, AM2, ESPCI, L1, M1, S1	AM1, INPG, S1, T1
ENS	INPG, P7, T3	INPG, P7, T3	INPG, P7, T3
G1	-	-	-
INPG	ENS, L1, M2, P7, S1, T3	AM1, ENS, P7, S1, T3	B1, ENS, P7, T3
L1	AM1, B1, INPG, S1		
P11	-	P6	-
P6	-	P11	-
$\mathbf{P7}$	ENS, INPG, T3	ENS, INPG, T3	ENS, INPG, T3
$\mathbf{S1}$	AM1, B1, INPG,L1, T3	AM1, AM2, B1, ESPCI, INPG, L1, M2	AM1, B1,ESPCI, L1
Т3	ENS, INPG, P7, S1	ENS, INPG, M2, P7	AM1, B1, ENS, INPG, P7

Table 11: Reference classes of the best ranked French universities in physics, with dominance relation and $\phi = 1$.

Table 12: Reference classes of the best ranked French universities in physics, with dominance relation and $\phi = .1$.

Citations Journal IF Rel JIF AM1 M2 B1, T3 AM2 B1, M2, S1 B1 B1 AM2, ESPCI, M2, S1 AM1, INPG, T3 ENS - P7 - ESPCI B1, M2, S1 - - G1 - - - INPG M2 - B1, T3 L1 - - - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 - P6 - - - P7 - ENS - P7 - ENS - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG				
AM1 M2 B1, T3 AM2 B1, M2, S1 B1 B1 AM2, ESPCI, M2, S1 AM1, INPG, T3 ENS - P7 - ESPCI B1, M2, S1 G1 - G1 - - - INPG M2 - B1, T3 L1 - - - P6 - - - P7 - ENS - P7 - B1, T3 - L1 - - - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG		Citations	Journal IF	Rel JIF
AM2 B1, M2, S1 B1 AM2, ESPCI, M2, S1 AM1, INPG, T3 ENS - P7 - ESPCI B1, M2, S1 - - G1 - P7 - - INPG M2 - B1, T3 - I1 - - B1, T3 - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 - - P6 - - - - P7 ENS - - - P7 FNS - - - P11 - - - - P6 - - - - P7 FNS - - - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	AM1		M2	B1, T3
B1 AM2, ESPCI, M2, S1 AM1, INPG, T3 ENS - P7 - ESPCI B1, M2, S1 - - G1 - - - INPG M2 - B1, T3 L1 - - - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 - P11 - - - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	AM2		B1, M2, S1	
ENS - P7 - ESPCI B1, M2, S1 - - G1 - - - INPG M2 - B1, T3 L1 - - - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 - P11 - - - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	B1		AM2, ESPCI, M2, S1	AM1, INPG, T3
ESPCI B1, M2, S1 G1 - - INPG M2 - B1, T3 L1 - - - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 - P11 - - - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	ENS	-	Ρ7	-
G1 - - INPG M2 - B1, T3 L1 - - - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 - P11 - - - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	ESPCI		B1, M2, S1	
INPG M2 - B1, T3 L1 - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 P11 - - - P6 - - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	G1	-	-	-
L1 - M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 P11 - - P6 - - P7 - ENS S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	INPG	M2	-	B1, T3
M2 B1, INPG AM1, AM2, B1, ESPCI, S1, T1 P11 - - P6 - - P7 - ENS S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	L1	-		
P11 - - P6 - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	M2	B1, INPG	AM1, AM2, B1, ESPCI, S1, T1	
P6 - - P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	P11	-	-	-
P7 - ENS - S1 T3 AM2, B1, ESPCI, M2 ESPCI T3 S1 M2 AM1, B1, INPG	P6	-	-	-
S1T3AM2, B1, ESPCI, M2ESPCIT3S1M2AM1, B1, INPG	$\mathbf{P7}$	-	ENS	-
T3 S1 M2 AM1, B1, INPG	$\mathbf{S1}$	T3	AM2, B1, ESPCI, M2	ESPCI
	T3	S1	M2	AM1, B1, INPG

	Citations	Journal IF	Rel JIF
AM2	M2	ENS, M2	M2
B1	ENS, Na1, R1		
G1	P7, T3	P7, S1	P7
L1	P5,S1,T3	P5,S1,T3	S1
M2	AM2	AM2	AM2
P11	-	-	-
P5	L1, S1	L1, S1, T3	S1, T3
P6	-	-	-
$\mathbf{P7}$	G1	G1	G1
S1	G1, L1, P5, T3	G1, L1, P5, T3	L1, P5, T3
Τ3	L1, S1	L1, P5, S1	P5, S1

Table 13: Reference classes of the best ranked French universities in all disciplines, with dominance relation and $\phi = 1$.

Table 14: Reference classes of the best ranked French universities in all disciplines, with dominance relation and $\phi = .1$.

	Citations	Journal IF	Rel JIF
AM2	M2	ENS	M2
B1	ENS		
ENS	B1	AM2	
G1	$\mathbf{S1}$	-	-
L1	P5	-	-
M2	AM2	-	AM2, ENS
P11	-	-	-
P5	L1	-	T3
P6	-	-	-
$\mathbf{P7}$	-	-	-
S1	G1	-	-
T3		-	P5



Figure 1: Adjusted dominance networks \vec{g}' among the top French research institutions built for dominance relations in fundamental biology, with $\phi = 1$ and with citations as the proxi for impact.



Figure 2: Adjusted dominance networks \vec{g}' among the top French research institutions built for dominance relations in fundamental biology, with $\phi = 1$ and with direct impact factor as the proxi for impact.



Figure 3: Adjusted dominance networks \vec{g}' among the top French research institutions built for dominance relations in fundamental biology, with $\phi = 1$ and with relative impact factor as the proxi for impact.



Figure 4: Adjusted dominance networks \vec{g}' among the top French research institutions built for dominance relations in fundamental biology, with $\phi = .1$ and with citations as the proxi for impact.



Figure 5: Adjusted dominance networks \vec{g}' among the top French research institutions built for dominance relations in fundamental biology, with $\phi = .1$ and with direct impact factor as the proxi for impact.



Figure 6: Adjusted dominance networks \vec{g}' among the top French research institutions built for dominance relations in fundamental biology, with $\phi = .1$ and with relative impact factor as the proxi for impact.